Promoting and Perfecting the *ars conjectandi*: On the 'charmed speculations' of Jacob Bernoulli and their contemporary reception in the Republic of Letters

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Long before Jacob Bernoulli's major work on probability theory, the Ars conjectandi, was published posthumously, news of his investigations spread across the networks of key mathematical players in Republic of Letters, including his younger brother Johann Bernoulli, their nephew Nicolaus Bernoulli, and the German mathematician and philosopher Gottfried Wilhelm Leibniz. These men were soon joined by Pierre Rémond de Montmort and Abraham de Moivre, both of whom became close friends with Nicolaus during the extended academic peregrination he undertook across Europe in 1712/13. It was mainly through the efforts of Nicolaus, who besides having studied mathematics in Basel under both of his uncles also trained as a lawyer, that the unfinished Ars conjectandi eventually came to be published in a form that could do justice to its author. Drawing on contemporary reports and correspondence, the paper seeks to throw new light on the publication of the 'art of conjecturing' at a time when key players in the debate over probability theory were beginning to forge ahead working on new problems and staking their claims to innovation and priority.

I. Introduction

At a time when relations among mathematicians in the Republic of Letters were already becoming strained through the emerging priority dispute over the discovery of the calculus, Johann Bernoulli (1667-1748), in early 1697, was able to convey news of a more encouraging kind to his friend and mentor Gottfried Wilhelm Leibniz (1646-1716).[1]

[1] I should like to thank participants of the Lyon workshop 'Je peux perdre, mais je gange toujours. Droit, espérance et probabilités à l'époque moderne', in particular the organizers Florian Reverchon and Filippo Contarini, for their comments on an earlier version of this paper. Sulamith Gehr of the Bernoulli-Euler-Zentrum at the Universitätsbibliothek Basel kindly directed me to a previously unpublished letter from de Moivre to Nicolaus Bernoulli. The following abbreviations are used: A, followed by series, volume, and page: G. W. Leibniz, *Sämtliche Schriften und Briefe,* Darmstadt, then Leipzig, and finally Berlin, ed. Prussian Academy of Sciences and its successors, 1923ff; GM: *Leibnizens Mathematische Schriften*, ed. Carl Immanuel Gerhardt, 7 vols, Berlin, then Halle 1849-1863; JABW: *Die Werke von Jakob Bernoulli*, ed. Joachim Otto Fleckenstein et al., 5 vols, Basel: Birkhäuser 1969-1999; JABB: *Der Briefwechsel von Jacob Bernoulli*, ed. David Speiser, Basel: Birkhäuser 1993; JOBB: *Der Briefwechsel von Johann I Bernoulli*, ed. Otto Spiess et al., 3 vols, Basel: Birkhäuser 1955-1992; NC: *The Correspondence of Isaac Newton*, ed. Herbert Westren Turnbull et al., 7 vols, Cambridge: Cambridge University Press 1959-1977.

Writing from Groningen, where he occupied the chair in mathematics, he points out that his elder brother Jacob (1655-1705) has already for many years been working on a major project called *Ars conjectandi*, in which not only all kinds of games were treated mathematically, but also questions of probability in all aspects of life were reduced to a sort of calculus.[2] Unfortunately, Johann had at the same time to concede that that he had absolutely no idea whether work on the *Ars conjectandi* would ever be completed, tacitly indicating thereby both the illness which had repeatedly impeded Jacob's ongoing investigations on probability theory as well as the estrangement which had by now existed between the two brothers for around five years.

It has to be said that Johann Bernoulli was well aware that Leibniz himself, since his sojourn in Paris between 1672 and 1676, had been interested in the application of algebraic techniques to the understanding of games. Partly this was a result of the German polymath's acquaintance with works by authors such as Blaise Pascal (1623-1662), Pierre Fermat (1607-1665), and Christiaan Huygens (1629-1695), and partly with a view to the possibilities such an approach might open for developing his pet project for eliciting new scientific discoveries, the *Ars inveniendi*.[3] And indeed when he returns to the topic in a later letter, Leibniz emphasizes the likely value of Jacob's book in illuminating the art of conjecture, while at the same time pointing out that already in his youth, notably in works dating from the late 1660s, he had concerned himself with its applications to jurisprudence and politics.[4] To underscore the depth and duration of his own deliberations, reflecting his legal training, Leibniz even notes that he has given the topic a name of his own devising: 'I call it the doctrine of degrees of probability'.[5] Establishing one's own claims to discoveries was, of course, common practice when it came to scientific correspondence, and there was no more convenient way of doing this than staking one's claim terminologically.

It is not necessary here to expand much further on Leibniz's remarks. Suffice it so say that in his reply to Johann Bernoulli he was referring in particular to two of his early works, the *Specimina juris* and the *Specimen demonstrationum politicarum*, both of which appeared in 1669. In the former, he sought to show how demonstrability could be

2 Johann Bernoulli to Leibniz, 20 February/[2 March] 1697, A III, 7, 309-310: '*nam frater meus jam a longis annis opus molitur quod* Artem conjecturandi *inscribet, ubi solum omnivarios ludos mathematice tractandi sed etiam alias in omni vitae genere probabilitates ad calculum revocandi modum traditurus est, nescio autem annon opus reliquerit imperfectum; saltem mea quae olim contuli quaeque ipse non spernenda judicavit, jam vix non expunget solita sua simultate agitatus*.'

3 See for example Leibniz to Johann Bernoulli, 29 January/[8 February] 1697, A III, 7, 293: '*Dici non potest quam multa ad Artem inveniendi utilia lateant in Ludis*.'; Leibniz to John Wallis, 12 October 1697, A III, 7, 586. See also Philip Beeley, Approaching Infinity: Philosophical consequences of Leibniz's mathematical investigations in Paris and thereafter, in: Mark Kulstad/Mogens Lærke/David Snyder (eds.), The Philosophy of the Young Leibniz, Stuttgart 2009, pp. 29-47.

4 Leibniz to Johann Bernoulli, 5/[15] March 1697, A III, 7, 329: '*Venit etiam in mentem rursus, quod de Domini fratris Tui* Arte conjecturarum *scripseras. Ea erit haud dubie non contemnenda. Ego quoque talia jam olim sum meditatus, praesertim in usum jurisprudentiae et politicae*.'

5 Ibid.: '*Voco doctrinam de gradibus probabilitatis*.'

brought to the presentation of legally complex cases,[6] while in the latter work, published under the pseudonym Georgius Ulicovius Lithuanus, he employed a mathematical approach to assessing the most reasonable outcome of the election of the Polish king.[7]

Coincidentally, around the same time as his exchange with Johann Bernoulli, Leibniz also mentions these youthful publications in his correspondence with the Scottish nobleman Thomas Burnett of Kemney (1656-1729) by way of an example of the civic applications of mathematics similar to the statistical investigations that had been conducted some thirty years earlier by William Petty (1623-1687):

> *I have strongly approved on other occasions of the thoughts of the late Mr Petty, who made clear the applicability of mathematics to politico-economic matters. I myself showed in a small book printed without my name in the year 1669 on the election of a king of Poland, at the request of an ambassador who had to go to Warsaw, that there is a kind of mathematics involved in the calculation of reasons, and that sometimes these must be added, sometimes multiplied together in order to get the correct sum. This is something that has not been noticed by the logicians.*[8]

II. Reports and rumours

Although Johann Bernoulli had scant knowledge of the most innovative aspects of his brother's work to disseminate in his network of correspondents, he remained for many years the most important source of information. Thus, incidentally, he had told his sometime pupil, Guillaume Marquis de l'Hôpital (1661-1704), who famously had paid him for instruction in Leibniz's infinitesimal calculus,[9] that Jacob had devised a theorem the proof of which he considered to be a greater accomplishment than if he had shown how to square the circle, since the latter, although a greater achievement, would have been of little practical use. Such a description was destined to generate interest on the part of l'Hôpital, further enhancing Johann Bernoulli's own standing in relation to him. It was, however, not a reflection of Johann's own assessment of the *Ars conjectandi*, but rather expressed the opinion of Jacob Bernoulli himself, who had made a corresponding entry in his *Meditationes*.[10] It refers specifically to his proof of the fundamental theorem con-

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6 The academy edition of *Specimina juris* is in A VI, 1, 366-430.

7 The academy edition of the *Specimen demonstrationum politicarum* is in A IV, 1, 3-98. Leibniz chose the pseudonym to correspond to his own initials.

8 Leibniz to Thomas Burnett of Kemney, 1/11 February 1697; Leibniz A I, 13, 551: '*J'ay fort approuvé autres fois les pensées de feu Mons. Petty, qui faisoit voir l'application des Mathematiques aux matieres oeconomico-politiques. Moy même, dans un petit livre imprimé l'an 1669 sans mon nom sur l'election d'un Roy de Pologne à la priere d'un Ambassadeur qui devoit aller à Warsovie, je fis voir, qu'il y a une espece de mathematique dans l'estime des raisons, et tantost il faut les adjouter, tantost les multiplier ensemble, pour en avoir la somme. Ce qui n'a pas esté remarqué des Logiciens*.' See also Leibniz to Johann Gröning, 24 December 1696/[3 January 1697], A III, 13, 452.

9 On this topic see Sandra Bella, La (Re)construction française de l'analyse infinitésimale de Leibniz, 1690-1706, Paris 2022, especially pp. 200-203, 209-224.

10 Jacob Bernoulli, *Meditationes*, Art 151a; JABW III, p. 88: '*N.B. Hoc inventum pluris facio, quam si ipsam*

tained in Part IV of the *Ars conjectandi*, namely, that the relative likelihood of various outcomes may be determined a posteriori when it is not known a priori:

> *When, however, it is the case and we eventually in this way obtain moral certainty (and that this actually is the case, I will show in the following chapter), then we can find the numbers of cases a posteriori almost just as exactly as if they were known to us a priori.*[11]

Having received the barest snippet of information from his teacher on what Jacob considered to be the most significant outcome of his work on the theory of conjecture, l'Hôpital was desperate to know more, as he demanded of Johann Bernoulli in his next letter, suggesting even that he should broach the topic with his elder brother:

> *Ask him, too, what is in his book de arte conjecturandi that proposition which he considers to be as important as the discovery of the quadrature of the circle? You see, Sir, that I continue to ask that you instruct me.*[12]

This was a wish Johann was unwilling and unable to fulfil. He had by all accounts misleadingly sought to persuade l'Hôpital that he knew more than he did, while effectively suggesting that his brother's most important insights were in fact his own. It says a lot about the contractual nature of the relationship between the French mathematician and Johann Bernoulli that l'Hôpital in his *L'analyse des infiniment petits pour l'intelligence des lignes courbes*, *the first systematic presentation of infinitesimal calculus*, published in 1696, barely acknowledges his exceptional debt to his former teacher, but notes instead the insights gained through the work of the two Basel brothers.[13]

It is not until 1703 that Leibniz takes up the topic of the 'estimation of probabilities' with Jacob Bernoulli himself, although the two men had by that time been in correspondence with one another for more than fifteen years. Knowing only what he had been able to elicit from Johann's correspondence some six years earlier, and aware of the fact that Jacob had up till then despite his evident capability published very little, namely a small number of articles in *Journal des Sçavans* and the *Acta Eruditorum*, he encourages him to complete his major work on the 'doctrine of estimating probabilities'. Interestingly, Leibniz only formulates this wish as a kind of afterthought in a completely revised postscript to his latest letter after having rejected another, much longer postscript concerning his introduction to questions of geometry during his time in Paris:

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circuli quadraturam dedissem, quod si maxime reperiretur, exigiu usus esset.'

11 Jacob Bernoulli, *Ars conjectandi* IV, §4, p. 226; JABW III, p. 249: '*At si id obtineat, acquiraturque tandem hoc pacto moralis certitudo (quemadmodum hoc etiam reapse fieri sequenti Capite ostendam) aeque propemodum exploratos habebimus a posteriori casuum numeros, acsi nobis a priori cogniti essent*.'

12 Marquis de l'Hôpital to Johann I Bernoulli, 8 December 1692, JOBB I, p. 160: '*demandez lui aussi quelle est cette proposition qui est dans son livre de arte conjecturandi dont il estimoit autant la decouverte que la quadrature du cercle. vous voyez, Monsieur, que ie continuë à vous prier de m'instruire*.'

13 The sole significant remark in this respect is found in the preface. See l'Hôpital, L'analyse des infiniment petits pour l'intelligence des lignes courbes, Paris: De l'imprimerie royale 1696, sig. eijv: '*Au reste je reconnois devoir beaucoup aux lumieres de Mrs. Bernoulli, sur tout à celles du jeune presentement Professeur à Groningue. Je me suis servi sans façon de leurs découvertes & de celles de M. Leibnis*.'

I hear that your doctrine of estimating probabilities (which I carry out a lot) is no less refined. I wish that someone would treat mathematically of the various kinds of games (of which there are examples in that doctrine). This would be delightful and at the same time useful, and not unworthy of you or any other serious mathematician.[14]

Alas, Jacob was unable to provide any hope that his book would soon be forthcoming. On the contrary, his long reply to Leibniz provides us with an early account of the tragic circumstances that had befallen him, most notably illness, and which would eventually mean that the fourth and most important part of the *Ars conjectandi* would remain incomplete. At the same time, his letter affords us insight into his deepfelt bitterness over the way he had been treated by his brother Johann, whom he correctly suspects to having been the source of Leibniz's information:

I would willingly know, most splendid Sir, from whom you have it that the doctrine of estimating probabilities has been perfected by me. It is true that over many years past I have been charmed by these speculations, which I scarcely think anyone else has thought about in this way. I was also inclined to draw up a certain tract on this topic, but repeatedly for years on end I have laid it aside, because of my natural listlessness, added to which is the incapacity caused by my frightful illness which as it gets worse makes it more difficult for me to write. […] I have nevertheless already completed the largest part of the book, but the most important part is still missing, in which I seek to apply the principles of the ars conjectandi to political, moral, and economic matters also: solved in a singular theorem in which the degree of difficulty is not small, and yet the degree of usefulness is by far the greatest.[15]

In his letter, Jacob also points out that some twenty years earlier he had set out his ideas on probability to his brother. And yet when Johann was questioned by the Marquis de l'Hôpital – in the letter cited above – he had made light of Jacob's investigations and instead passed off his brother's most significant findings as his own. There is little reason to doubt the truth of what Jacob reports, but unfortunately the evidence is missing: Johann Bernoulli's letter to l'Hôpital responding to the Marquis's question has not survived.

14 Leibniz to Jacob Bernoulli, mid-April 1703; Leibniz A III, 9, 289: 'P.*S. Audio a TE doctrinam de aestimandis probabilitatibus (quam ego magni facio) non parum esse excultam. Vellem aliquis varia ludendi genera, (in quibus pulchra hujus doctrinae specimina) mathematice tractaret. Id simul amoenum et utile foret, nec TE aut quocunque gravissimo Mathematico indignum.*' See Anders Held, A History of Probability and Statistics and their Applications, before 1750, New York 1990, p. 253.

15 Jacob Bernoulli to Leibniz, 3 October 1703; Leibniz A III, 9, 359: '*Scire libenter velim, Amplissime Vir, a quo habeas, quod Doctrina de probabilitatibus aestimandis a me excolatur. Verum est, me a plurimis retro annis hujusmodi speculationibus magnopere delectari, ut vix putem, quenquam plura super his meditatum esse. Animus etiam erat, Tractatum quendam conscribendi ad hac materia; sed saepe per integros annos seposui; quia naturalis meus torpor, quem accessoria valetudinis meae infirmitas immane quantum auxit, facit ut aegerrime ad scribendum accedam; […] Absolvi tamen jam maximam Libri partem, sed deest adhuc praecipua, qua artis conjectandi principia etiam ad civilia, moralia et oeconomica applicare doceo, soluto eum in finem singulari quodam Problemate, quod difficultatis commendationem non parvam, utilitatis longe maximam habet.*'

III. The Law of Large Numbers

The findings referred to were essentially those mentioned above and which, as Jacob now reveals to Leibniz, consist above all in the law of great numbers: that with any particular probability the result depends on the number of cases in which it might or might not obtain. Decisively, Jacob points out to Leibniz that 'we can find the number of cases a posteriori almost as exactly as if they were known to us a priori'.[16]

Nor does Jacob stop there. He proceeds to explain to Leibniz how increasing the number of observations increases the probability that the relation of the number of favourable observations to unfavourable ones eventually exceeds any given degree of certainty whatever. He compares the nature of this approach to a true relation to that of an asymptote and concludes by emphasizing that estimations reached in this way are entirely suitable for making decisions in civil society:

> *The relation between the number of cases which I achieve in this way is correct and natural (genuinam); which suffices to make conjectures in such contingent matters for everyday civil use no less scientifically tractable than a game of dice, wherein alone, I suppose, the prudence of the politician consists.*[17]

Jacob draws here directly on what he has set out in the *Ars conjectandi*. The corresponding passage in this work reads thus:

> *Rather, one must also consider other things, about which perhaps no-one up to now has even given thought to. It remains to be investigated, namely, if by steadily increasing the observations the probability will also grow that the number of favourable observations compared to the number of unfavourable observations reaches the true relation, namely to the extent that this probability finally exceeds any given degree of certainty, or if rather the problem , so to speak, has its asymptote, that is to say, whether a certain degree of certainty of having found the true relation of cases has been obtained, which even by increasing the number observations never can be exceeded, for example, that we can never beyond 1/2, 2/3 or 3/4 degree of certainty achieve the assurance to have found the true relation of cases.*[18]

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16 Ibid., p. 360: '*Nam si hoc sit, actum erit de nostro conatu explorandi numeros casuum per experimenta: sin illud, aeque certo rationem illorum a posteriori indagabimus, atque si nobis a priori cognita esset*.' See Held (fn. 14), p. 225.

17 Ibid.: '[…] *rationem inter numeros casuum, quam hoc pacto obtineo, legitimam et genuinam esse; quod in usu vitae civilis sufficit, ad conjecturas nostras in quavis materia contingente non minus scientifice dirigendas atque in ludis aleae, in quo solo omnem Politici prudentiam consistere puto*.'

18 Jacob Bernoulli, *Ars conjectandi* IV §4, p. 225; JABW III, p. 249: '*Ulterius aliquid hic contemplandum superest, quod nemini fortassis vel cogitando adhucdum incidit. Inquirendum nimirum restat, an aucto sic observationum numero ita continuo augeatur probabilitas assequendae genuinae rationis inter numeros casuum, quibus eventus aliquis contingere & quibus non contingere potest, ut probabilitas haec tandem datum quemvis certitudinis gradum superet: an vero Problema, ut sic dicam, suam habeat Asymptoton, h. e. an detur quidam certitudinis gradus quem nunquam excedere liceat, utcunque multiplicentur observationes, puta, ut nunquam ultra semissem, aut 2/3, aut 3/4 certitudinis partes certi fieri possimus, nos veram casu-*

Such claims on the part of Jacob Bernoulli were destined to provoke a lively debate with Leibniz who had his own idiosyncratic ideas on questions relating to necessity and contingency: ideas that were deeply rooted in his philosophy, that is to say, the doctrine of monads. Thus, in the course of their sporadic correspondence which continued up to Jacob's death in 1705, Leibniz would emphasize the usefulness of 'the estimating of probabilities', while at the same time suggesting that in the case of juridical and political matters there was not so much a need for a precise calculus as for the accurate enumeration or recapitulation of all the relevant circumstances. Precisely herein lay in his view the difficulty with Jacob's approach. Then, for him contingent events always depended on an infinite number of circumstances that simply could not be ascertained precisely by finite experiences. In this respect, Leibniz was able to draw on his knowledge as a diplomat and lawyer, arguing that it was generally not possible to find cases where the same circumstances prevailed and where one case could therefore serve as a model for another. In consequence, he was convinced that an a posteriori estimation such as Jacob conceived was impossible. Moreover, even if repetition of circumstances were possible, there would still remain a fundamental problem in principle, as he points out in a letter written towards the end of 1703:

> *Since we estimate probabilities empirically by experiences of sequencies, you ask whether there is nonetheless a way by which a perfect estimation can be obtained. You say that you have discovered this. The difficulty which lies in it appears to me to be that contingent things or that which depends on infinite circumstances cannot be determined by a finite number of experiences; nature has namely its own customary laws, resulting from causes, but only for the most part.*[19]

Having been informed by Jacob earlier of his younger brother's deceitful behaviour, Leibniz took care to remove any suspicion that Johann Bernoulli had been the source of his knowledge of the *Ars conjectandi*, but could only remember his knowledge to have come from elsewhere.[20] Maintaining scientific discourse in the Republic of Letters often required a good sense of diplomacy.

In his reply, written in the spring of the following year, Jacob Bernoulli points out that probability theory is important in legal affairs with regard to insurance, annuities on lives, marriage contracts, and so on.[21] Once more, he explains that his theorem pro-

um rationem detexisse.' See Held (fn. 14), p. 249.

19 Leibniz to Jacob Bernoulli, 3 December 1703; Leibniz A III, 9, p. 406-407: '*Cum Empirice aestimamus probabilitates per experimenta successuum, quaeris an ea via tandem aestimatio perfecta obtineri possit. Idque a Te repertum scribis. Difficultas in eo mihi inesse videtur, quod contingentia seu quae ab infinitis pendent circumstantiis per finita experimenta determinari non possunt; natura quidem suas habet consuetudines, natas ex reditu causarum, sed non nisi ὡς ἐπὶ τὸ πολὺ.*' See also Leibniz, *Nouveaux Essais IV*, 16, §9; A VI, 6, 464-465.

20 Ibid.: '*Haec a Te tractata non primum a Domino Fratre Tuo, sed aliunde me discere memini*.'

21 Jacob Bernoulli to Leibniz, 20 April 1704; A III, 9, p. 478: '*Quod Doctrina de probabilitatibus aestimandis in materiis juridicis non sola circumstantiarum enumeratione, sed eodem illo ratiocinio et calculo indigeat, quo alias in sortibus aleatorum computandis uti solemus, docent me variae quaestiones de Assecurationibus, de Reditibus ad vitam, de Pactis dotalibus, de Praesumptionibus, aliaeque; quemadmodum suo tempo-*

vides him with the means of determining unknown probabilities with moral certainty – and that precisely this suffices for practice in civil affairs. For him, estimating probabilities in juridical matters did not come down simply to estimating circumstances, but to a form of calculus, namely such as one was otherwise accustomed to use in computing the chances of throwing a specific number of dice. He proceeds to provide an empirical example, similar to one contained in the *Ars conjectandi*, which as he now tells Leibniz he had shown his brother some twenty years earlier and received his approval:

> *I suppose that hidden in a certain urn are some small stones, white and black, and that the number of white stones is double the number of black stones, but that you do not know this ratio and want to determine it by experiment. Accordingly, you draw one stone after another (replacing each stone after you have drawn it before you choose the next one so that the number of stones in the urn is not reduced) and you observe the white or black stone you have chosen. I say now that (with the two assumed relations approaching as far as you want the double ratio, the one larger, the other smaller, say, 201:100 and 199:100) the scientific limit of the number of observations which you carry out being ten times or a hundred times or a thousand times, as appears to you more probable, so far the ratio of the number of turns, in which you choose a white stone to the number of turns in which you choose a black stone, falls within these limits of the double ratio of cases 201:100 and 199:100 than outside. Indeed, you can thus have the moral certainty that the ratio discovered by experiment will approach the true double ratio as much as one wants.*[22]

IV. Completing the *Ars conjectandi*

Following Jacob's death in August 1705, Leibniz advised his heirs to allow the Basel mathematician Jacob Hermann (1678-1733), a former pupil of Jacob's, to sort out his papers and to order the manuscript of the still incomplete *Ars conjectandi*.[23] His widow subsequently commissioned Hermann with this task, he thus gaining the possibility of copy-

re liquido ostendam.'

[22] Ibid.: '*Pono in urna quadam reconditos esse calculos aliquot, albos et nigros, et numerum alborum esse duplum numeri nigorum, Te autem nescire hanc rationem, et experimentis illam determinare velle. Educis itaque calculum unum post alterum (reponendo singulis vicibus illum quem eduxisti, priusquam sequentem eligis, ne numerus calculorum in urna minuatur) et observas, albus an ater sit quem elegisti. Dico jam, quod (assumtis duabus rationibus rationi duplae quantumvis propinquis, una majore, minore altera, puta 201:100 et 199:100) scientifice determino numerum observationum quem si instituas, decies, aut centies aut millies etc. probabilius tibit fiat, rationem numeri vicium, quibus album eligis, ad numerum vicium quibus eligis nigrum, intra quam extra hos limites rationis duplae 201:100 et 199:100 casuram; adeo ut tandem moraliter certus esse possis, rationem per experimenta deprehensam verae rationi duplae quantumvis proxime accessuram.*' The corresponding passage is found in *Ars conjectandi* IV, §4, p. 226; JABW III, p. 249.

[23] See Leibniz to Jacob Hermann, 21 September 1705; GM IV, pp. 284-285. On Jacob Hermann's relations to Jacob Bernoulli see Fritz Nagel, Jacob Hermann – Skizze einer Biographie, in: Fritz Nagel/Andreas Verdun (eds.), 'Geschickte Leute, die was praestiren können…' Gelehrte aus Basel an der St. Petersburger Akademie der Wissenschaften des 18. Jahrhunderts, Aachen 2005, pp. 55-75.

ing some of the pieces and above all of providing contemporary mathematicians with a better idea of what his *Nachlass* contained. By the end of October, Hermann was able to report to Leibniz rather optimistically that the

> *Art of Conjecturing, as he called it, is now only a little away from being completed, and he would have put the finishing touches to it, if he had survived his fate by just a couple of months. The whole work is divided into four parts, of which the first is a small tract by Huygens on calculations in games of chance complemented by his own distinguished commentary. The second contains the doctrine of permutations and combinations. The third sets out the use of the preceding doctrine in various games of chance and games of dice. The fourth part finally treats of the use as well as the application of the aforementioned in civil, moral, and economic matters.*[24]

This was indeed an accurate summary of the contents which broke down thus in the finally published version:

- *Tractatus Hugenii De ratiociniis in Ludo Aleae, cum Annotationibus Jacobi* Bernoullii; pp. 2-71.
- *Doctrina de permutationibus & combinationibus*; pp. 72-137
- *Usus praecedentis doctrinae in variis sortitionibus & ludis aleae*; pp. 138-209
- *Usus & applicatio praecedentis doctrinae in civilibus, moralibus & oeconomicis*; pp. 210-239

> *As appendix, the work included the Lettre à un Amy, sur les Parties du Jeu de Paume, in which Bernoulli presents certain propositions on possible outcomes of the ball game by that name; 35pp.*

It was thanks to efforts of Hermann that a rough outline of the main contents of the *Ars conjectandi* became more widely known and were also mentioned, amongst others, in the *éloge* for Jacob Bernoulli delivered to the Académie royale des sciences by the Paris mathematician Joseph Saurin (1659-1737).[25] Unfortunately, Saurin's suggestion that the *Ars conjectandi* be prepared for publication by Johann Bernoulli was roundly rejected by

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[24] Jacob Hermann to Leibniz, 28 October 1705; GM IV, pp. 285-286: '*Ars, quam vocabat, Conjectandi parum ab omnimoda perfectione abest, ultimamque accepisset manum, si vel paucis duntaxat mensibus fato suo supervixisset. Totum opus in quatuor dividitur partes, quarum prima Hugenianum tractatulum de ratiociniis in ludo aleae cum additis ad eum insignibus notis propriis complectitur. Secunda continet doctrinam de Permutationibus et Combinationibus. Tertia usum doctrinae praecedentis in variis sortitionibus et ludis aleae explicat. Pars tandem quarta usum quoque tradit et applicationem praecedentium in Civilibus, Moralibus et Oeconomicis.*'

[25] [Joseph Saurin], Eloge de M. Bernoulli, cy-devant Professeur de Mathematique à Bâle, in: Journal des Sçavans (February 1706), pp. 81-89. Saurin's final remarks refer to the *Ars conjectandi*: '*Il y a lieu d'esperer que quelque main amie & habile ajoûtera à un Traité si curieux ce qui peut y manquer; mais quand on le donneroit tel qu'il est, il sera toûjours beaucoup de plaisir au Public, sur-tout si avec les Tables Gnomoniques universelles, qu'on dit être en état de voir le jour, on y joint ce qu'on trouvera parmi les papiers de cet illustre Geometre, de plus digne de sa reputation & de la curiosité des Sçavans. C'est ce qu'ils attendant en particulier des soins de M. Bernoulli le jeune, qui a toûjours fait paroître un grand zele pour l'utilité publique, & pour l'avancement des Sciences, & à qui le memoire d'un tel frere doit être chere.*' (p. 89). See Held (fn. 14), p. 223-224.

the author's widow for reasons that don't need to be repeated. The distrust of Johann Bernoulli ran deep in his brother's family.[26]

It would seem that little happened in the field of probability theory that was deemed worthy of reporting in the Republic of Letters during the next few years. But then, in May 1708, at the end of a letter to Leibniz principally concerned with the latest developments in the priority dispute with Newton, Johann Bernoulli slipped in a noteworthy postscript:

> *A man by the name of Monsieur de Montmort, who formerly wrote to you from Groningen under the name de Remond, canon of Nôtre Dame in Paris, is currently having a book printed on games of chance.*[27]

Despite his extensive correspondence network, Leibniz had not heard anything about Pierre Rémond de Montmort's (1678-1719) work before this time. However, prompted by Bernoulli's note on the French aristocrat, Leibniz reiterated in his reply his desire that Jacob's deliberations on probability theory should not be lost to posterity; indeed, he would wish that choice parts of the *Ars conjectandi* be published sometime.[28]

For the time being nothing could be done on that account, but in a letter to Leibniz a year later, in April 1709, Bernoulli introduces another mathematician of his family, Nicolaus Bernoulli (1687-1759), who would eventually play a decisive role in the publication of his uncle's most important work. Nicolaus was the son of another brother of Jacob and Johann I Bernoulli, the Basel painter and alderman Nicolaus Bernoulli (1662-1716). He had studied in Basel under both of his uncles: first under Jacob, and then with Johann, his successor on the chair in mathematics. While under the tutelage of Jacob, he had been able to consult his unpublished manuscripts, including those of the *Ars conjectandi* and would later make important contributions to probability theory of his own. Although he undoubtedly benefitted from his family heritage, Nicolaus nonetheless encountered a significant hurdle in his path to an academic career. Because of the shortage of suitable posts for mathematicians in the Swiss confederation – and indeed in Europe more generally – he found it necessary to devise a fall-back. For this reason, he additionally studied jurisprudence and in 1709 succeeded in gaining a doctorate in both laws with a dissertation on the legal applications of probability theory, the *Dissertatio inauguralis mathematico-juridica de usu artis conjectandi in jure* (Figure 1). This was in many ways a natural continuation of the kind of work Jacob had been doing, and not least for this reason Nicolaus was in many ways predestined to take on the editorial role he later carried out.

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26 See Johann Bernoulli's remarks on this topic in his letter to Leibniz, dated 1 September 1708; GM III, pp. 837-838.

27 Johann Bernoulli to Leibniz, 16 May 1708; Leibniz GM III, p. 827: '*Un nommé Mr. De Montmort qui vous a autrefois écrit à Groningue sous le nom de Remond, Chanoine de notre Dame de Paris, fait actuellement imprimer un livre sur les jeux de hazard.*'

28 Leibniz to Johann Bernoulli, 27 June 1708; GM III, p. 836: 'N*on memini aliquid audire de Montmortio, qui de ludis fortunae librum parat. Vellem hoc argumentum bene tractaretur. Sed et Domini Fratris Tui, piae memoriae, meditationis de talibus vellem non perire. Credo enim aliquid in illis fore non spernendum. Et omnino optarem selecta ex ipso aliquando in lucem proferri.*'

Johann uses his letter to inform Leibniz in considerable detail of Nicolaus's legal studies, knowing well that the German mathematician and philosopher had himself studied law and had published a number of tracts on philosophical and logical aspects of jurisprudence. Indeed, he compares Nicolaus's efforts to apply probability theory to law with the origins of his own concern for the applications of mathematics. But before doing so he provides Leibniz with the latest news on Montmort's scholarly endeavours, although he makes clear at this time that he does not expect much to come out of them. Despite already having mentioned the sometime pupil of Pierre Malebranche (1638-1715) in his previous letter, Johann evidently felt the need to re-introduce Montmort, perhaps simply to emphasize better the achievements of his nephew. He could not know at this point that Nicolaus Bernoulli and Montmort would go on to collaborate productively with each other on games of chance and probability theory:

> *A certain Rémond de Montmort recently wrote that he will send me his book whose title is Essay d'Analyse sur les jeux de hazard. However, I doubt that he will treat of this matter sufficiently enough. The son of my brother has prepared for the press an inaugural dissertation on a similar matter, namely the use of the art of conjecturing in law, in which he has undertaken to treat various customary questions in law, especially about absentees taken to be dead persons, who later turn out to be alive. Thus, just as I once did with medicine, he now undertakes to apply our mathematics not unproductively to jurisprudence, which at any rate will be something new and unusual among lawyers. As soon as the dissertation leaves the press, I will send it to you by the most convenient means you indicate to me.*[29]

Despite his initial misgivings over the likely quality of Montmort's book, Johann Bernoulli reached to a different opinion once the promised copy came into his hands. Not only did he soon thereafter lend his copy of *Essay d'analyse* to his nephew but also encouraged him to study it carefully. In a long letter to Montmort of March 1710, in which Johann Bernoulli himself discussed the work in considerable detail, he also enclosed a copy of his nephew's comments. Despite the productive mathematical contents, there is an unmistakable poignancy in his words. Against the background of the havoc wreaked by the War of Spanish Succession and loss felt at the departure of Jacob Hermann to take up the mathematics chair in Padua three years earlier, he paints a bleak picture of the state of mathematics in his native country, excepting only for the time being the promising career of his nephew:

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29 Johann Bernoulli to Leibniz, 15 April 1709; Leibniz GM III, p. 842: '*Quidam Remundus de Montmort scripsit nuper mihi se ad me missurum librum suum, cujus Titulus Essay d'Analyse sur les jeux de hazard; dubito autem an bene satis tractaverit hanc materiam. Fratris mei Filius ad prelum parat Dissertationem inauguralem Juridicam de simili materia, nempe De usu artis conjectandi in jure, ubi tractandas suscipit quaestiones varias in Jure agitare solitas, praecipue circa absentes pro mortuis habendos, reditus item vitales etc. adeo ut, quam ego olim ad Medicinam, ille nunc ad Jurisprudentiam non inutiliter applicare instituat Mathesin nostrum, quod quidem apud Jurisconsultos (qui hunc tractandi modum insuper habent) aliquid novi et insoliti erit. Ubi prelum evaserit dissertatio, eam quoque at Te mittam, modo mihi commodam mittendi viam indices.*'

> *I hear with pleasure that mathematics despite the miseries of the war is flourishing more and more, and are even gaining honour in France. Here in our country, we cannot boast the same happiness. Since the departure of M. Hermann, I know no-one apart from my nephew and very few others from whom one can expect any great progress in these sciences, which being considered insufficient to earn one's bread are neglected as things unfertile and scarcely of use.*[30]

After Leibniz, through the efforts of Johann Bernoulli, had been able to read Nicolaus's dissertation and see for himself the young man's ability, he suggested to Johann that his nephew would be ideally placed to carry out 'an incredibly useful task' to the scientific public by pursuing Jacob's work on the estimation of probabilities further, and indeed bringing it to completion.[31] Leibniz's opinion carried weight. However, by the time his letter containing this suggestion arrived in Basel, Nicolaus had already embarked upon what would turn out to be a long European journey taking him first to France, England, the Low Countries, and then back to France. It was not until he first stopped in Paris that Nicolaus was able to learn of Leibniz's proposal which was conveyed to him by a letter from his uncle, Johann Bernoulli.[32]

V. Nicolaus Bernoulli in London

By the end of September 1712, Nicolaus arrived in London, where he soon became acquainted with Abraham de Moivre (1667-1754), a Huguenot mathematician, who since the revocation of the Edict of Nantes was living in London where he became a close friend of Isaac Newton (1642-1727). De Moivre was himself working on aspects of the theory of probabilities and its applications, and the previous year had published his tract on the measurement of chance, *De mensura sortis*, in the *Philosophical Transactions of the Royal Society*.[33] Nicolaus was naturally drawn to de Moivre's work, a copy of which

30 Johann Bernoulli to Pierre Rémond de Montmort, 17 March 1710, in: Montmort, Essay d'Analyse sur les jeux de hazard. Seconde édition revûe & augmentée de plusieurs Lettres, Paris 1713, pp. 297-298: '*J'entens avec plaisir que les Mathematiques, malgré les miseres de la guerre fleurissent de plus en plus, & deviennent même en honneur en France. Ici dans nos pays nous ne pouvons pas nous vanter du même bonheur. Depuis le départ de M. Herman je ne sçai personne, excepté mon neveu & très peu d'autres, dont il faille esperer de grands progrès dans ces sciences, lesquelles étant considerées comme n'être pas* de pane lucrando, *on les néglige comme des choses seches & peu utiles.*' Nicolaus Bernoulli's remarks on the first edition of Montmort's *Essay d'analyse* are contained in the second edition pp. 299-303.

31 Leibniz to Johann Bernoulli 6 June 1710; GM III, p. 850: '*Si Dominus Agnatus Tuus, juri dans operam, urgeat et perficiat coepta Fratris Tui, Domini Jacobi Bernoullii, circa aestimationes probabilitatum, faciet rem utilissimam.*'

32 See Johann Bernoulli to Leibniz, 12 August 1710; GM III, p. 852: '*Agnato meo, qui nunc Parisiis agit, perscripsi monitum Tuum de perficiendis coeptis circa aestimationes probabilitatum.*'

33 Abraham de Moivre, De mensura sortis, seu, de probabilitate eventuum in ludis a casu fortuito pendentibus, in: Philosophical Transactions No. 329 (January, February and March 1711), pp. 213-264. On the background to de Moivre's work see David Bellhouse, Banishing Fortuna: Montmort and De Moivre, in: Journal of the History of Ideas 69 (2008), pp. 559-581; esp. pp. 570-572.

the author gave to him at one of their frequent meetings,[34] and through studying it discovered an alternative solution to the so-called *Spieldauer*-Problem, or problem of duration of play, that was subsequently published in the *Philosophical Transactions* for 1714.[35]

In other ways, too, Nicolaus was soon able to establish his reputation in the English metropolis. After Newton had heard from de Moivre that Nicolaus Bernoulli had found and corrected a mistake in a series expansion contained in Book II, Proposition 10 of the *Principia*, Newton quickly satisfied himself of his error and the validity of the correction.[36] He then lost no time in wanting to make the acquaintance of the young Swiss mathematician and a number of meetings including dinners at Newton's invitation followed. In 1714, Nicolaus Bernoulli was admitted to the Royal Society on the proposal of Edmond Halley (1656-1742), immediately after his uncle Johann Bernoulli was likewise made a fellow.

These honours, it should be pointed out, were partly based on a misleading attribution of the discovery that can largely be put down to Nicolaus himself. The discovery was in fact made by Johann Bernoulli, who had begun systematically collecting errors in the *Principia* in 1710, and had communicated it to Leibniz in August of that year.[37] Johann later assumed that his nephew had seen his comments on the *Principia* back in Basel before elucidating its true source in greater depth.[38]

Once Johann Bernoulli had provided Montmort with his extensive comments on his *Essay d'analyse*, in March 1710, he allowed his nephew to take over his role in correspondence with the French mathematician, Nicolaus tactfully explaining this change through Johann's preoccupation with other matters.[39] In fact, it fitted well with Nicolaus's travel plans in which Paris figured centrally. Thus, after he had embarked upon his *peregrinatio academica*, Nicolaus kept Montmort informed of his itinerary and looked forward to discussing with him 'more agreeably' in person those mathematical topics that had previously been the subject of their letters, as he wrote from London in October 1712.[40] They had important things to discuss. In the same letter, Nicolaus informed Montmort of a discovery he had made on the basis of an article for divine providence published in the *Philosophical Transactions*. The article concerned was the work of the Scottish physician

34 Nicolaus Bernoulli to Pierre Rémond de Montmort, 11 October 1712; Montmort (fn. 30), p. 375: '*J'ai le plaisir de voir ici souvent M. de Moivre qui m'a fait présent de son Livre De Mensura Sortis.*'

35 Nicolaus Bernoullis, Solutio Generalis Problematis XV. proposit a D. de Moivre, in tractatu de Mensura Sortis inserto in Actis Philosophicis Anglicanis No. 329. pro numero quocunque Collusorum, in: Philosophical Transactions No. 341 (December 1714), pp. 133-144.

36 See Abraham de Moivre to Johann Bernoulli, 18 October 1712; *Der Briefwechsel*, pp. 270-271. See also Nicolaus Bernoulli to I. Newton, 20 May 1717; NC VI, p. 288.

37 Johann Bernoulli to G. W. Leibniz, 12 August 1710; GM III, pp. 854-855.

38 See I. Newton to Nicolaus Bernoulli, c. 1 October 1712; NC V, pp. 348-350; I. Newton to Roger Cotes, 31 March 1713; NC V, p. 400.

39 Nicolaus Bernoulli to Rémond de Montmort, 26 February 1711; Montmort (fn. 30), p. 308.

40 Nicolaus Bernoulli to Rémond de Montmort, 11 October 1712; Montmort (fn. 30), p. 375: '*J'espere bientôt repasser en France, & d'avoir l'honneur de m'entretenir avec vous sur ces matieres plus agréablement que nous n'avons fait jusqu'ici dans nos Lettres.*'

John Arbuthnot (1667-1735), and was based on birth records of infants in London for each of the eighty-two years from 1629 to 1710.[41] On examining these records, Arbuthnot found a near exact balance between male and female births, which could not be accounted for by pure chance: innumerable throws of a suitably configured die, Arbuthnot argued, would yield an infinitely small probability or at least one less than any assignable fraction could not be the effect of chance but instead was necessarily the result of divine providence: 'From whence it follows, that it is Art, not Chance, that governs.'[42]

As Nicolaus tells Montmort, in view of the approval he had encountered for the argument for divine providence in discussions earlier in Holland and now in England, he felt obliged to refute the argument and to prove that there is a high probability that the number of number of males and females every year comes within much narrower limits than those that had been observed since eighty years in succession. He continues:

> *You know well Monsieur it would be something ridiculous to want to prove that it is very probable that the number of boys will be exactly the same as the number of girls; but that the ratio between the number of the one and that of the other will very probably approach one of equality is something of which I am sure you will be persuaded. I found on examining the catalogue of infants born in London from 1629 up to and including1710 that there are more males than females, & that taking their mean the proportion of males to females is very nearly that of 18 to 17, just a little larger. From this I conclude that relation of the probability for the birth of a boy to the probability for the birth of a girl is around that of 18 to 17, & that therefore out of 14,000 infants, which is approximately the number of infants which are born each year in London there will be about 7,200 males & 6,800 females. Now, the year in which the largest number of males, in relation to that of females, are born, was the year 1661, in which 4,748 males & 4,100 females are born; & the year in which there was the smallest number of males in relation to that of females, is the year 1703, in which 7,765 males & 7,683 females are born. I say that these limits are so large that one can wager at least by 300 to 1 that out of 14,000 infants the number of males & of females will more likely fall within these limits than outside them.*[43]

[41] John Arbuthnot, An Argument for Divine Providence, taken from the constant Regularity observ'd in the Births of both Sexes, in: Philosophical Transactions 328 (for October, November, and December 1710), pp. 186-190.

[42] Ibid., p. 189.

[43] Nicolaus Bernoulli to Pierre Rémond de Montmort, 11 October 1712; Montmort (fn. 30), p. 374: '*Vous sentés bien, Monsieur, que ce seroit une chose ridicule, si l'on vouloit prouver qu'il est fort probable que le nombre des garçons sera justement égal au nombre des filles; mais que la raison entre le nombre des uns & des autres approchera fort près de la raison d'égalité, est ce dont je crois que vous ferés persuadé. J'ai trouvé en examinant le Catalogue des enfans nés à Londres depuis 1629 jusqu'à 1710 inclusivement, qu'il y a plus de mâles que de femelles; & qu'en prenant un milieu, la raison des mâles aux femelles est fort près de la raison de 18 à 17, un peu plus grande; d'où je conclus que la probabilité, pour qu'il naisse un garçon, est à la probabilité pour qu'il naisse une fille environ comme 18 à 17, & qu'ainsi entre 14000 enfans, qui est à peu près le nombre des enfans qui naissent par an à Londres, il y aura environ 7200 mâles & 6800 femelles. Or*

This argument would provide an important pretext for Nicolaus to revisit the work of his deceased uncle Jacob Bernoulli. After leaving London, Nicolaus proceeded as planned via the Low Countries back to Paris, but if he was expecting to meet Montmort there, he would have been sorely disappointed. With the French mathematician presumably instead residing at the family château in Montmort-Lucy, Bernoulli had no alternative than for the time being to continue his epistolary exchange. Importantly, he now returned to his argument concerning Arbuthnot's article on the record of infant births in London, not only setting it out again, but also now taking the opportunity of sending Montmort a copy of the catalogue on which it was based (Figure 2). At the end of his letter, Nicolaus pointed out that through his deliberations he had been able to verify the crucial conclusion Jacob Bernoulli had drawn in the *Ars conjectandi* regarding the law of large numbers:

> *I remember that my late uncle demonstrated something similar in his tract De arte conjectandi, which is currently at the press in Basel, namely, that if one wants to discover by often repeated experiments the number of cases in which a certain result comes about or not, one can increase the number of observations in such a way that eventually the probability that we have discovered the true relation between the number of cases is greater than any given probability. When his book comes out, we will see if in these kinds of matters I have found an approximation which is just as correct as the one he made.*[44]

Nicolaus encouraged Montmort to examine the catalogue and see if he arrived at the same conclusions. He effectively initiated thereby the close collaboration between the two mathematicians that would soon be continued in person at Montmort's château, where Nicolaus Bernoulli, residing as a guest, worked with Montmort for some three months on preparing the second edition of *Essay d'analyse*, while jointly pursuing further their investigations. The resultant volume, published in 1713, would contain a substantial amount of the correspondence exchanged by the two men between February 1711 and November 1713 along with the initial letter to Montmort from Johann Bernoulli and Montmort's reply which effectively brought that intellectual commerce into being. It documents an important part of the fruitful and flourishing discussion on probability theory that took place at the time.

l'année où il est né le plus grand nombre de mâles, par rapport à celui des femelles, a été l'année de 1661, dans laquelle il est né 4748 mâles & 4100 femelles; & l'année où il y avoit le plus petit nombre de mâles par rapport à celui des femelles, est l'année 1703, dans laquelle il est né 7765 mâles & 7683 femelles. Je dis que ces limites sont si grands, qu'on peut parier au moins plus de 300 contre 1 qu'entre 14000 enfans le nombre des mâles & des femelles tombera plûtôt entre ces limites que dehors.'

44 Nicolaus Bernoulli to Pierre Rémond de Montmort, 23 January 1713; Montmort (fn. 30), p. 393: '*Je me souviens que feu mon Oncle a démontré une semblable chose dans son Traité De Arte conjectandi, qui s'imprime à présent à Bâle, sçavoir, que si l'on veut découvrir par les experiences souvent réiterées le nombre des cas par lesquels un certain évenement peut arriver ou non, on peut augmenter les observations en telle maniere qu'enfin la probabilité que nous ayons découvert le vrai rapport qu'il y a entre les nombres des cas, soit plus grande qu'une probabilité donnée. Quand ce Livre paroîtra nous verrons si dans ces sortes de matieres j'ai trouvé une approximation aussi juste que lui.*'

VI. De Moivre's *De mensura sortis*

By the time Nicolaus Bernoulli arrived in London, in 1712, de Moivre had already been in correspondence with his uncle for more than eight years. Their letters enable us to form a clearer picture not only of how Nicolaus was received in London's scientific circles, but also how his work on probability theory impacted contemporary discussion more widely. In particular, we see how over a short period of time due to shared scientific interests the correspondence between de Moivre and Johann Bernoulli intersects with that between Nicolaus Bernoulli and Pierre Rémond de Montmort. Thus, we find Johann Bernoulli writing to de Moivre in June 1712, thanking him for the copy of *De mensura sortis* he had recently received. While characteristically noting that he had himself solved many of the problems dealt with in de Moivre's *Traité sur le hazard*, Bernoulli praises, too, the very abstract approach of Huguenot mathematician and the high degree of generality achieved in his solutions:

> *Monsieur de Montmort will see that many of his particular cases are absorbed into the wide extent of the generality of yours, just as little streams lose themselves in the ocean into which they flow.*[45]

Montmort was of a different opinion, but with four highly competent and ambitious mathematicians working on similar problems, it was not surprising that minor disagreements arose, even if these were usually short-lived. Thus, in the dedicatory letter to Francis Robartes (c. 1649-1718), with which de Moivre prefaced his *De mensura sortis*, he placed emphasis on the greater level of generality he had achieved in his treatment of problems of chance than his predecessors Christiaan Huygens (1629-1695) and Pierre Rémond de Montmort:

> *Huygens was the first that I know who presented solutions for this kind of problems, which a French author has very recently illustrated handsomely with various examples; but these distinguished gentlemen do not appear to have employed that simplicity and generality which the nature of the matter demands.*[46]

Montmort naturally felt the need to respond to this criticism and used the opportunity of a letter to Nicolaus Bernoulli to do so,[47] knowing full well that what he wrote was likely to reach the author of *De mensura sortis*. Indeed, this was the case. Although Montmort's letter did not reach Nicolaus until after he had left England, he duly made de Moivre a copy of that part of it which concerned him.[48] The French aristocrat had complained to

45 Johann Bernoulli to Abraham de Moivre, 23 November 1712, in: Karl Wollenschläger (ed.), Der mathematische Briefwechsel zwischen Johann I Bernoulli und Abraham de Moivre, Basel 1933, p. 275: '*M. de Montmort verra que plusieurs de ses regles et solutions particulieres seront absorbées dans l'étendue de la généralité des vôtres, comme les petits ruisseaux se perdent dans l'Océan, dans lequel ils se déchargent.*'

46 de Moivre (fn. 33), p. [214]: '*Hugenius, primus quod sciam regulas tradidit ad istius generis Problematum Solutionem, quas nuperrimus autor Gallus variis exemplis pulchre illustravit; sed non videntur viri clarissimi ea simplicitate ac generalitate usi fuisse quam natura rei postulabat.*'

47 Pierre Rémond de Montmort to Nicolaus Bernoulli, 5 September 1712; Montmort (fn. 30), p. 362.

48 Abraham de Moivre to Johann Bernoulli, 17 December 1712; Der mathematische Briefwechsel (fn.45),

Nicolaus that for all his efforts towards generality, de Moivre had only solved the easiest problems in the *Essay d'analyse*, not those at the end which were more difficult.[49] Interestingly, de Moivre did not thank Nicolaus for this kindness for well over a year, presumably because he subsequently focused on the problems Montmort had raised.[50]

As de Moivre informed Johann Bernoulli, it was during a discussion with his friend Robartes over the *Essay d'analyse* – that is to say, the first edition of Montmort's work – that he was moved to develop the ideas set out in *De mensura sortis*.[51] He tells him, too, that he held back in pointing out some mistakes of various kind in Montmort's arguments so as not to cause him trouble: 'I truly hold him in esteem and hope that we will become good friends.'[52] Indeed, despite various quarrels along the way, the two men parted as such after Montmort visited de Moivre in London in 1715.[53]

VII. Printing *Ars conjectandi*

It was de Moivre's account of the circumstances under which he wrote *De mensura sortis* that prompted Johann Bernoulli to first let him know about the efforts to get his brother's *Ars conjectandi* printed. In a letter written soon after Nicolaus's arrival in London, he paints a decidedly bleak picture of what the learned public could expect to see published in the name of his late brother, aligning the person overseeing publication with those Plato would not admit to his academy on account of their being ignorant of geometry:

> *My nephew who had the opportunity to go deeply into these matters, having been obliged to write an inaugural dissertation, will have told you that one has begun to print the posthumous work of my late brother, which he entitled Ars conjectandi. However, it is very imperfect and hardly half of it completed, the main and most interesting part which should deal with morals not having been carried out because of the death of the author. Since the heirs of my late brother are having it printed without consulting me about it, I did not want to interfere or offer my services in order that it be printed correctly. Meanwhile, he who has undertaken the correction does not understand the manner of writing, is unable to recognize the outlines, and is igno-*

p. 279.

49 Pierre Rémond de Montmort to Nicolaus Bernoulli, 5 September 1712; Montmort (fn. 30), p. 362: '[…] *vous trouverés que son travail se borne presqu'entierement à résoudre d'une maniere plus generale que je n'ai fait, les questions les plus simples & les plus faciles qui sont dans mon Livre*.' See Bellhouse (fn. 33), p. 573.

50 Abraham de Moivre to Nicolaus Bernoulli, 3 March 1714; Universitätsbibliothek Basel, UBH L Ia 22:2 Nr. 180a: '*Apres la lettre que Vous me fites l'honneur de m'ecrire de Hollande dans laquelle etoit Inclus un Manuscript d'une lettre de Monsieur Montmaur, dont je Vous remercie icy tres humblement quoy qu'un peu tard, je me sentis excité a travailler a ce Probleme, Monsieur De Montmaur avoit jugé qu'un des Corollaires que j'avois tiré de mon Probleme de la Poule à trois, etoit hardy* […].'

51 Abraham de Moivre to Johann Bernoulli, 18 October 1712; *Der mathematische Briefwechsel* (fn.45), p. 272.

52 Ibid.: '*je me suis bien gardé de relever quelques fautes de M. de Montmort d'une maniere diverse, et qui ait pu lui faire de la peine; j'ai véritablement de l'estime pour lui et j'espere que nous serons bons amis.*'

53 Abraham de Moivre, Miscellanea analytica de seriebus et de quadraturis, London 1730, p. 249.

> *rant of geometry. I am telling you this so you are not shocked when you see a monster appear bearing the name of my late brother. It is a pity that this book did not reach its completion. One has told me (for I have not read the manuscript) that it should contain something of extraordinary value: effectively the project my brother revealed to me a good twenty years ago promised many curiosities on the art of conjecturing. It is possible that one will see some features in that which appears, provided the impression is not too disfigured by the innumerable number of faults that have escaped the corrector due to ignorance.*[54]

Meanwhile, Nicolaus had begun a correspondence with Leibniz during his journey abroad. In his first letter, sent from London, he apologizes for not having written earlier to thank the German mathematician and philosopher for the encouragement he had given to his work on probabilities.[55] Now, writing from England, he was able to provide Leibniz with news slightly more optimistic than what de Moivre had received from his uncle: Nicolas notes, too, that printing of the *Ars conjectandi* has already begun, and imperfectly, but he holds out hope that on his return to Basel he will be able to supplement the work with what is missing so long as the heirs of his late uncle agree to entrust him with his papers.[56] The fact he was also Jacob's nephew would have given him grounds for optimism.

Nicolaus neglected to say precisely when he was expecting to arrive back in the city of his birth – and with good reason. As we have seen, after he left London, he continued his *peregrinatio academica*, concluding the tour at the beginning of April 1713 after a long stay at Montmort's château. While there, his host wrote to Johann Bernoulli describing Nicolaus's industriousness, while also being not entirely modest about his own capabilities:

> *Your nephew is incredibly learned and very untiring. I cannot work for more than two hours at a go, while he works for six hours without getting ti-*

[54] Johann Bernoulli to Abraham de Moivre, 23 November 1712; *Der mathematische Briefwechsel*, (fn.45) pp. 275-276: '*Mon neveu qui a eu l'occasion d'approfondir cette matiere, ayant été obligé d'écrire une dissertation inaugurale, vous aura dit, qu'on a commencé à imprimer l'ouvrage posthume de feu mon frere, qu'il a intitulé 'Ars conjectandi', quoiqu'il soit très imparfait, et à peine achevé jusqu'à la moitié, le principal et le plus curieux qui devoit rouler sur la morale n'ayant pu être exécuté à cause de la mort de l'auteur: comme les héritiers de feu mon frere le font imprimer sans me consulter là-dessus, je n'ai pas voulu m'y ingérer, ni offrir mes soins qu'il soit exactement imprimé; cependant celui qui a entrepris la correction, n'entend pas la maniere, c'est un aveugle à discerner les contours, c'est un* αγεωμέτρητος*; je vous le dit afin que vous ne vous scandalisiez pas, quand vous verrez paroître un monster qui portera le nom de feu mon frere; c'est dommage que ce livre ne soit pas à sa perfection. On me dit (car je n'ai ne pas lû le manuscrit) qu'il devoit contenir quelque chose d'extraordinaire; effectivement le projet que mon frere m'en fit voir, il y a bien 20 ans, promettoit bien des curiosités sur l'art de conjecturer: on en verra peut-être quelques traits dans ce qui paroitra, pourvû que l'impression ne soit pas trop défigurée par un nombre innombrable de fautes qui s'y glisseront par l'ignorance du corrrecteur.*'

[55] Nicolaus Bernoulli to G. W. Leibniz, 25 October 1712; GM III, p.979.

[56] Ibid., p. 981: '*Basileae imprimitur Patrui mei Tractatus posthumus de Arte Conjectandi, sed imperfectus; si post reditum meum in Patriam heredes defuncti mihi schedas ejus confidere velint, tractabo supplere, quae desunt.* '

> *red. He has proposed to me many fine problems which I have undertaken with a great deal of honour and happiness. These will be augmentations to my book. I believe you will be happy with a complete tract I am writing on combinations. I do not believe that one will be able to take this topic much further.*[57]

Indeed, apart from including his *Traité de combinaisons* along with a large part of the correspondence he had exchanged with Nicolaus Bernoulli between 1711 and 1713 in the second edition of his *Essay d'analyse*, Montmort benefitted greatly from his discussions with the young Swiss mathematician in developing further his own ideas on probability theory, as reflected in many of the problems the book contains.

Montmort also notes Nicolaus had informed him of the immanent publication of the *Ars conjectandi* and lends his weight to the scholarly demand that the work be completed from surviving manuscripts.[58] In the end Nicolaus's hope of being able to see once more the papers of his uncle was thwarted by Jacob Bernoulli's heirs. His son Nicolaus, now in charge, refused to allow anyone to see them. The rift in the family was evidently irrepairable. All Nicolaus could do was to try to reconstruct from memory those missing parts of the *Ars conjectandi* he had seen earlier during his studies. But he was at least also able to address the chief fear Johann Bernoulli had set out to de Moivre, who on seeing the publication praised it as sustaining perfectly well the reputation of the author.[59] Back home in Basel, he set about correcting the imperfectly prepared printed sheets, and, likewise at the request of the local printers Thurneysen brothers, added a short preface.[60] As a small token of good will, his cousin Nicolas had made this request, too.

VIII. Conclusion

Against the backdrop of his elder brother's illness, Johann I Bernoulli served for in the early 1700s as the most well-informed, although not always trustworthy, source for Leibniz and others of information on Jacob Bernoulli's *Ars conjectandi*. When Leibniz eventually entered correspondence with the work's author, their discussion on probability theory soon faltered due to fundamentally diverging concepts of conditionality. Contemporary work on the topic was instead promoted by others, namely Abraham de Moivre, Pierre Rémond de Montmort, and above all Nicolaus Bernoulli, who during his *peregri-*

57 Pierre Rémond de Montmort to Johann Bernoulli, 5 March 1713, Universitätsbibliothek Basel, UBH L Ia 665 Nr. 7: '*Mr. vostre neveu est furieusement sçavant et tres infatigable. Je ne peux travailler deux heures de suitte, il en travaille six sans estre las. il m'a proposé plusieurs problemes fort jolis et je m'en suis tiré avec assez d'honneur et de bonheur, ce sera des Augmentations pour mon livre, je crois que vous serez content d'un traitté complet que je donne sur les combinaisons. Je ne crois pas qu'on puisse pousser plus loin cette matiere.*'

58 Ibid.: '*Mr. vostre neveu m'appris que le livre de feu Mr. vostre frere paroistroit bientost. On feroit bien plaisir aux Geometres si l'on donnoit en meme temps ce que trouvera de plus complet dans ses manuscripts.*'

59 Abraham de Moivre to Nicolaus Bernoulli, 3 March 1714; Universitätsbibliothek Basel, UBH L Ia 22:2 Nr. 180a: '*c'est un Ouvrage qui soutient parfaitement bien la reputation de son Auteur.*'

60 Nicolaus Bernoulli to Pietro Antonio Michelotti, 29 August 1713, Universitätsbibliothek Basel, UBH L Ia 21:1, f. 123-124r.

natio academica effectively became the torch bearer of his uncle's scientific legacy. Not only was Nicolaus able to draw on memory, having had first-hand knowledge of Jacob's work during his studies, and particularly during his productive collaboration with Montmort push those ideas further, but also he was the principal source of news on ongoing efforts in Basel to get the *Ars conjectandi* into print. The long-held hope that by granting him access to Jacob's surviving papers it would be possible to complete the all-important fourth part of the work in the end remained unfulfilled. The story of the *Ars conjectandi* thus at once became a tragic combination of uncompleted mathematical brilliance on the one side and destructive family drama on the other.

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