Euler’s Ether Pressure Model of Gravitation

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Introduction

During the 17th century the notion arose that the weights and the free fall of bodies towards the Earth as well as the motion of the planets around the Sun might be caused through one and the same physical mechanism. The search for the reason of terrestrial attraction and planetary motion produced a wide variety of theories and models, particularly in the century between 1640 and 1740. The leading concepts were associated with the vortex theory by Descartes and the theory of universal gravitation by Newton. However, all theories failed to solve one crucial issue, namely to explain the physical cause of attraction and gravitation.

The vortex theories yielded „reasonable“ mechanisms to explain the phenomena qualitatively, but they lacked of adequate mathematical laws to determine and predict orbits and trajectories of heavenly and terrestrial bodies. The inverse square law allowed correct quantitative predictions of the motion of celestial bodies, *e.g.*, the Moon’s apsidal motion, as well as the figure of the Earth or the tides. It turned out to be a powerful and successful tool in mathematical astronomy, *e.g.*, celestial mechanics, and in geophysics. In the late 1740s the advent of perturbation theory initialized the triumphant advance of what later became established as „Newton’s theory of universal gravitation“. For the majority of natural philosophers the „theory“ or „principle“ of universal gravitation became a synonym for the inverse square law. The true nature and cause of gravitation, however, remained unexplained. The seemingly absurd notion of matter acting upon other matter at a distance had become „an ordinary inconceivability“, accepted as an irreducible principle. Some „fundamentalists“ clearly recognized this unsatisfactory situation and continued to look for a mechanical explanation of gravitation. Most prominent were the two Swiss scientists Leonhard Euler (1707 – 1783) and George-Louis Le Sage (1724 – 1803). Both created theories of gravitation based on the principle of action by contact caused by a supposed ubiquitous ether. In Euler’s model gravitation was produced by the distribution of ethereal pressure. In Le Sage’s model it was realized by the impulses of ethereal particles moving omnidirectionally with high velocities. Both models had their deficiencies and imperfections, and neither model was completed and published in a consistent way. What was put foreward was not generally accepted.

Euler’s theory of gravitation was the central theme of a few papers written in the 19th and 20th century. Each contains important aspects, but none treats the topic comprehensively or points out the essential ideas in Euler’s theory. (Isenkrahe, 1881) published his paper when the search for the nature of gravitation was again in the center of scienctific interest. He judged Euler’s theory with respect to its usefulness for the contemporary research. His contribution is therefore not a historiographical analysis. (Pulte, 1989) recognized that Euler‘s discovery of the principle of least action might have played an important role in the formation of Euler‘s gravitational model. (van Lunteren, 1991) gives an overview of the concepts of gravitation in the 18th and 19th century and puts Euler’s theory in the context of the contemporary ether theories of gravitation without analyzing it thoroughly, however. (Wilson, 1992) is the most complete treatise of our topic. He discussed the development of Euler’s theory of gravitation and, though not as sensitively as Pulte did, Euler‘s metaphysical foundations of mechanics, which he judged as „impossible“. He attributed the „hypothetical aethereal physics“ as „Eulerian“ without recognizing the novelties inherent in Euler’s gravitational model.

Euler’s Gravitational Model

In 1727 Euler seemed to accept publicly the „attractive force of Newton“, because he did not doubt „that all bodies by their nature attract one another“. There are reasons which motivated Euler to change his mind and to consider a model to explain gravitation physically (first using the concept of ethereal vortices, then ethereal pressure) and mathematically (in accordance with the inverse square law and Kepler’s laws) as well. We skip the individual steps Euler made in developing his theory of gravitation and refer to the „milestones“ published in (Euler, 1744, 1746a, 1746b, 1746c, 1746d, 1751, 1752). Let us focus on his final and most relevant treatise on the subject, probably written during the years 1755-1758. The *Anleitung zur Naturlehre* (Euler, 1862), often regarded as Euler‘s main work on natural philosophy and on his metaphysical foundations of rational mechanics, actually is mainly devoted to one topic: matter and ether, both considered as a continuum, and their relation causing all phenomena associated with structure, elasticity, and motion of bodies, including gravity, as well as their definition using principal properties.

For Euler, empty space is not acceptable. Instead of this hypothesis he postulates the existence of an omnipresent, extremely thin and subtle continuous „matter“ which is permanently compressed and which is characterized by an extremely high elasticity and an extremely low density. This medium is Euler’s ether, and he derives gravity from ethereal pressure. The ether would not be able to push bodies through impacts of its particles, because the resulting force would depend critically upon the density of the ether, which was known to be extremely low, as one might conclude from the unhindered motion of the celestial bodies. Euler postulates a disequilibrium of the ether in the neighbourhood of a celestial body, resulting in a pressure diminution inversely proportional to the distance from the body’s center. Let (Fig. 1) the pressure in the general ether be h. A test body with base area aa, length b, and weight P will, if placed at a distance x from the center C of a celestial body A, be pressed down to C by a force $rr/xx \cdot P$, where r is the radius of A. This force corresponds to the inverse square law because of the pressure gradient. The downwards force

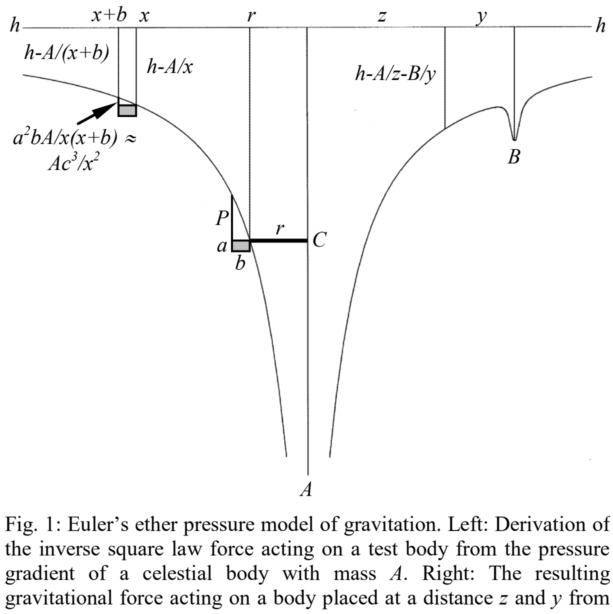

Fig. 1: Euler’s ether pressure model of gravitation. Left: Derivation of the inverse square law force acting on a test body from the pressure gradient of a celestial body with mass A. Right: The resulting gravitational force acting on a body placed at a distance z and y from the celestial bodies A and B, respectively.

acting on the top of the body will be $aa\cdot(h-A/(x+b))$, and the upwards force acting on the bottom of the body will be $aa\cdot(h-A/x)$, which is smaller, so that the net downwards force will be given by $aabA/x(x+b)$. Because b is much smaller than x, it may be neglected. Euler (incorrectly) concluded from the theory of hydrodynamics that the resulting force acting on the body would be proportional to the total volume aab associated with its „true size“ (Fig. 2) and thus would be proportional to its mass. This volume consists of particles of homogeneous matter, defined as those parts of the body impermeable for the ether. If A is assumed to be the mass of the celestial body and $c^3=aab$ the mass of the test body, Euler actually obtains the correct gravitational force Ac^3/x^2. The resulting gravitational force acting on a test body by several masses is determined through the same procedure, as illustrated on the left hand side of Fig. 1. Generally, the ether pressure of a test body placed at the distances z from A, y from B, x from C, etc. is given by $h-A/z-B/y-C/x-\ldots$etc. (see right hand side of Fig. 1).

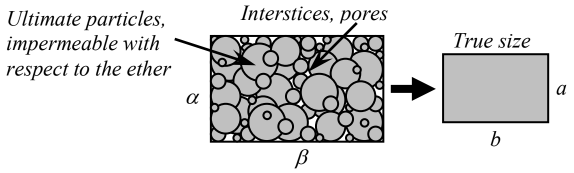

Fig. 2: Euler's explanation of the relation between volume and mass of a body. A body with base area $\alpha\alpha$ and length β consists of ultimate particles of homogeneous matter which are impermeable with respect to the ether, and of the interstices or pores between these particles. The true size of the body corresponds to the volume aab it would have if no pores were present.

The Scientific Relevance of Euler's Model

Euler used this result as an additional argument for the superiority of the pressure theory when compared to explanations based on impact, because the effect of the latter critically depends on the shape of the body. When Clairaut proved the inverse square law to be correct, Euler might have become more confident of his model. On the other hand, Euler was aware of the fact that his model was insufficient and deficient. He failed, *e.g.*, to explain the reason for the high pressure of the ether and its diminution near massive bodies. He knew that his model would be judged as an anachronism by the scientific community which accepted universal gravitation, whatever it was, as a „law“. He furthermore was aware of the critique by Le Sage. These facts may explain why Euler never published his theory. He just could not bring his theory into a final form. From the historical point of view, however, Euler's attempt to find a physical explanation of gravitation deserves attention and respect not only because of its role as a peculiar „outsider theory“ but, in particular, because of attractive characteristics of his model.

(Pulte, 1989) has shown that Euler might have interpreted planetary motion as a forced motion within the ether. This motion would be governed by the principle of least action discovered by Euler in 1743. Euler thus recognized that in real nature there is a principle different from central forces which may be used to explain the motion and the trajectory of a body. This principle seemingly justifies his gravitational model and his choice of primary properties of matter (which we did not discuss here). Euler's notion of ether, considered as a continuum, is a real novelty. By not accepting empty space, Euler replaced it, not verbally but implicitely, by his ether. Gravitation thus was explained by the local structure of space, *i.e.*, the distribution of ethereal pressure caused by massive bodies. Euler's theory of gravitation may thus be interpreted as the first attempt to describe universal gravitation in terms of a „field“ theory, as we would call it today. There are no „field equations“ or anything similar of that kind in Euler's theory. It had no impact on his famous and invaluable contributions to celestial mechanics or even on his perturbation theory. His model should be judged, in retrospective, as an intelligent attempt to explain gravitation in a way it was never done before.

Conclusions

Euler's ether pressure model of gravitation was regarded as out of date already by his contemporaries. It was a first fruitful attempt to explain gravity using mechanical principles. His model is what we would call today a field theory of gravitation. Euler was able to express his ideas only mechanically and not in terms used later on. When characterizing his ether, Euler sometimes compares it to a loaded fixed beam having a tension like a spring, assuming its original form when unloaded. Today, we would illustrate this analogy in a more general way. Fig. 3 shows a membrane, representing the three-dimensional space, loaded by three masses, representing celestial bodies. General Relativity often refers to this analogy. The geometric interpretation of Einstein's theory, that „the masses tell space, how it has to curve, and the space tells the masses, how they have to move“, might be translated into Euler's terms in the following way: „The masses tell ether pressure, how it has to be distributed, and the distribution of ether pressure tells the masses, how they have to move“. This parable illustrates what might have been Euler's thoughts.

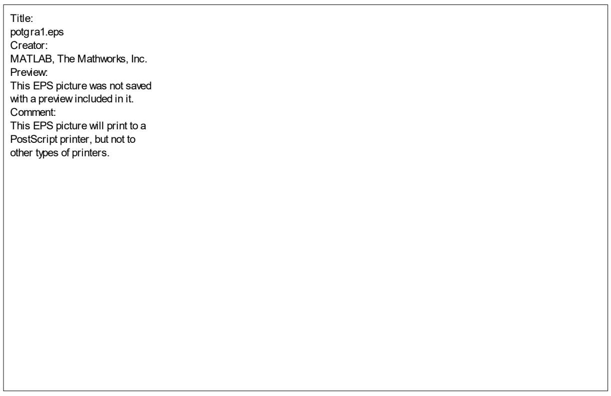

Fig. 3: Modern interpretation of Euler's idea of the distribution of ether pressure illustrated by a membrane, representing the three-dimensional space, and loaded by masses, representing celestial bodies.

References

Euler, L. (1744): *LEOO* I, **24**, 298-308, (E65).
Euler, L. (1746a): *LEOO* II, **8**, 27-53, (E82).
Euler, L. (1746b): *LEOO* III, **5**, 1-45, (E88).
Euler, L. (1746c): *LEOO* II, **31**, 195-220, (E89).
Euler, L. (1746d): *LEOO* III, **1**, 6-15, (E91).
Euler, L. (1751): *Opuscula varii argumenti* **3**, 1-53, (E109), *LEOO* III, **10**, (in preparation).
Euler, L. (1752): *LEOO* II, **5**, 109-131, (E181).
Euler, L. (1862): *LEOO* III, **1**, 16-178, (E842).
Isenkrahe, C. (1881): Euler's Theorie von der Ursache der Gravitation. *Zeitschrift für Mathematik und Physik (Historisch-literarische Abtheilung)*, **26**, 1-19.
Lunteren, F. v. (1991): *Framing Hypotheses*. Conceptions of gravity in the 18th and 19th centuries. Ph. D. thesis, University of Utrecht, Utrecht.
Pulte, H. (1989): *Das Prinzip der kleinsten Wirkung und die Kraftkonzeptionen der rationalen Mechanik*. Eine Untersuchung zur Grundlegungsproblematik bei Leonhard Euler, Pierre Louis Moreau de Maupertuis und Joseph Louis Lagrange. *Studia leibnitiana*, Sonderheft 19. Stuttgart, Steiner.
Wilson, C. A. (1992): Euler on action-at-a-distance and fundamental equations in continuum mechanics. In: HARMAN, P. M. / SHAPIRO, A. E. (eds.): *The investigation of difficult things*. Cambridge, UP, 399-420.