DIE SPRINGBRUNNEN VON SANSSOUCI

HANSPETER KRAFT

Das Scheitern des Brunnenprojektes von Friedrich dem Grossen in seinem Schloss Sanssouci in Potsdam diente bis in die Neuzeit als Symbol für das Auseinanderklaffen von Theorie und Praxis und gipfelte in der Feststellung, dass der Mathematiker Euler eben ein zweitklassiger Physiker war und wenig Ahnung von den Anwendungen hatte. Wir verdanken es Michael Eckerts sorgfältiger Recherche in [Eck02], mit der er diese Legenden korrigierte und das Scheitern des Projektes auf die Tatsache zurückführen konnte, dass der König unfähige Praktiker pfuschen liess und vor den hohen Ausgaben für ein so aufwändiges Projekt zurückschreckte. Mit der Erlaubnis des Autors Eckert zitieren wir frei aus seiner Arbeit (siehe pdf im Anhang).

Euler und Sanssouci: Zitate

Leonhard Euler (1707-1783) ist als mathematisches Genie bekannt. Er wird auch als Pionier der theoretischen Mechanik gerühmt. Die Gleichungen, die die Bewegung eines Kreisels beschreiben, werden „Euler-Gleichungen" genannt. Ein anderer Satz von „Euler-Gleichungen" wird in der Hydrodynamik verwendet um den Fluss idealer Flüssigkeiten zu beschreiben. Gleichzeitig soll Euler an einem praktischen hydrodynamischen Problem gescheitert sein: Der König von Preussen, Friedrich der Grosse, beauftragte Euler mit der Berechnung der Hydraulik für einen Springbrunnen in seinem Park Sanssouci in Potsdam, doch das Projekt kam nie zustande, weil Eulers Theorie für diese Aufgabe unbrauchbar erschien. Dieses Scheitern in Sanssouci wurde zum Symbol für die Spaltung zwischen Theorie und Praxis im 18. Jahrhundert und bis in die Neuzeit.

> *„M. Euler paroissoit quelquefois ne s'occuper que du plaisir de calculer, et regarder le point de Mécanique ou de Physique, qu'il examinoit, seulement comme une occasion d'exercer son génie et de se livrer à sa passion dominante. ... Nous conviendrons que le premier reproche n'étoit pas sans fondement, nous avouerons que M. Euler le Métaphysicien, ou même le Physicien, n'a pas été si grand que le Géomètre; ..."* (Condorcet 1783 [Con83])

> *„The physical universe was an occasion for mathematics to Euler, scarcely a thing of much interest in itself; and if the universe failed to fit his analysis it was the universe which was in error."* (E. T. Bell 1937 [Bel37])

> *„Der geniale Mathematiker Euler war zweitklassig als Physiker. ... Aufschluss gefunden hätte er in den Schriften des von ihm verehrten Leibniz."* (A. Hermann 1991 [Her91, S. 80–81])

> *„When Euler applied his equations to design a fountain for Frederick the Great of Prussia, it failed to work ... Unfortunately, he omitted the effects of friction, with embarrassing practical consequences."* (S. Perkovitz 1999 [Per99, p. 38])

Eulers angebliches Versagen in Sanssouci wurde nie von Wissenschafts- und Technologiehistorikern untersucht – trotz der häufigen Bezugnahmen darauf in Kommentaren zur Kluft zwischen Hydrodynamik und Hydraulik, idealen und realen Flüssigkeiten und anderen Formen der Kluft zwischen Theorie und Praxis. Ein Physiker führte den Fall des „Mathematikers Euler" an, um den Unterschied zwischen jemandem zu veranschaulichen, der sich mit idealen und realen Flüssigkeiten befasst:

> *„Leider hat er die Auswirkungen der Reibung ausser Acht gelassen, mit peinlichen praktischen Konsequenzen. Als Euler seine Gleichungen anwendete, um einen Brunnen für Friedrich den Grossen von Preussen zu entwerfen, funktionierte es nicht."*

Ein Technikhistoriker führte Eulers Scheitern als Beispiel für die vergeblichen Versuche im Zeitalter der Aufklärung an, Mathematik auf praktische Probleme anzuwenden:

> *„Friedrich der Grosse befahl 1749, dass Euler eine Maschine zur Wassergewinnung berechnen sollte, die für die Brunnen von Sanssouci verwendet werden sollte. Aber Eulers Theorie war für praktische Zwecke nicht anwendbar.*

Auch mehrere Jahre nach dem Scheitern hat Friedrich der Grosse in einem Brief an Voltaire über Euler gespottet:

> *„Je voulus faire un jet-d'eau en mon Jardin; le Ciclope Euler calcula l'éffort des roües, pour faire monter l'eau dans un bassin d'où elle devoit retomber par des Canaux, afin de jaillir à Sans-Souci. Mon Moulin a été éxécuté géométriquement, et il n'a pu élever une goute d'eau à*

> *Cinquante pas du Bassin. Vanité des Vanités; Vanité de la géométrie."* (FREDERICK II an VOLTAIRE, 25. Januar 1778)

Aber was sind die Beweise für EULERs angebliches Scheitern? Was war die Natur des hydrodynamischen Problems und wie war es mit dem praktischen Problem des Baus eines Springbrunnens verbunden? Was war das zeitgenössische Wissen über Hydrodynamik und Hydraulik im 18. Jahrhundert, auf dem EULER seine Arbeit in Sanssouci gründete?

Die Baugeschichte, Eulers Auftrag und seine Warnungen

In der Baugeschichte von Potsdam [Man89] findet man eine ausführliche Beschreibung der Bemühungen um diesen Springbrunnen und von den geplatzten Rohren. Im September 1749 übermittelte EULER dem König seine Berechnungen, wobei er in einem Brief an MAUPERTUIS auch erste Überlegungen zur Rohrdicke anstellte.

In einem weiteren Brief im Oktober 1749 stellt EULER klar fest, dass die Kapazität der Pumpen viel zu gross ist und die Rohre unweigerlich platzen werden, falls man nicht den Durchmesser der Rohre oder die Transport-Höhe stark reduziert.

> *„La véritable cause de ce fâcheux accident consistoit uniquement en ce que la capacité des pompes étoit trop grande, et à moins qu'on ne la diminue très considérablement, ou en diminuant leur diamètre ou leur hauteur, ou le nombre des jeux qui repond à un tour de moulin, la machine ne sera pas en état de fournir une seule goûte d'eau dans le réservoir."* (EULER an MAUPERTUIS, 21. Oktober 1749)

Mehr Details findet man im Vortrag *„Euler und die Springbrunnen von Sanssouci"*, welchen MICHAEL ECKERT anlässlich der Feier zum 300sten Geburtstag von LEONHARD EULER im September 2007 in Basel gehalten hat und welchen wir hier mit seiner Erlaubnis abdrucken dürfen.

Literatur

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[Man89] Heinrich Ludwig Manger, *Baugeschichte von Potsdam*, vol. 1–3, Friedrich Nicolai, Berlin, 1789.

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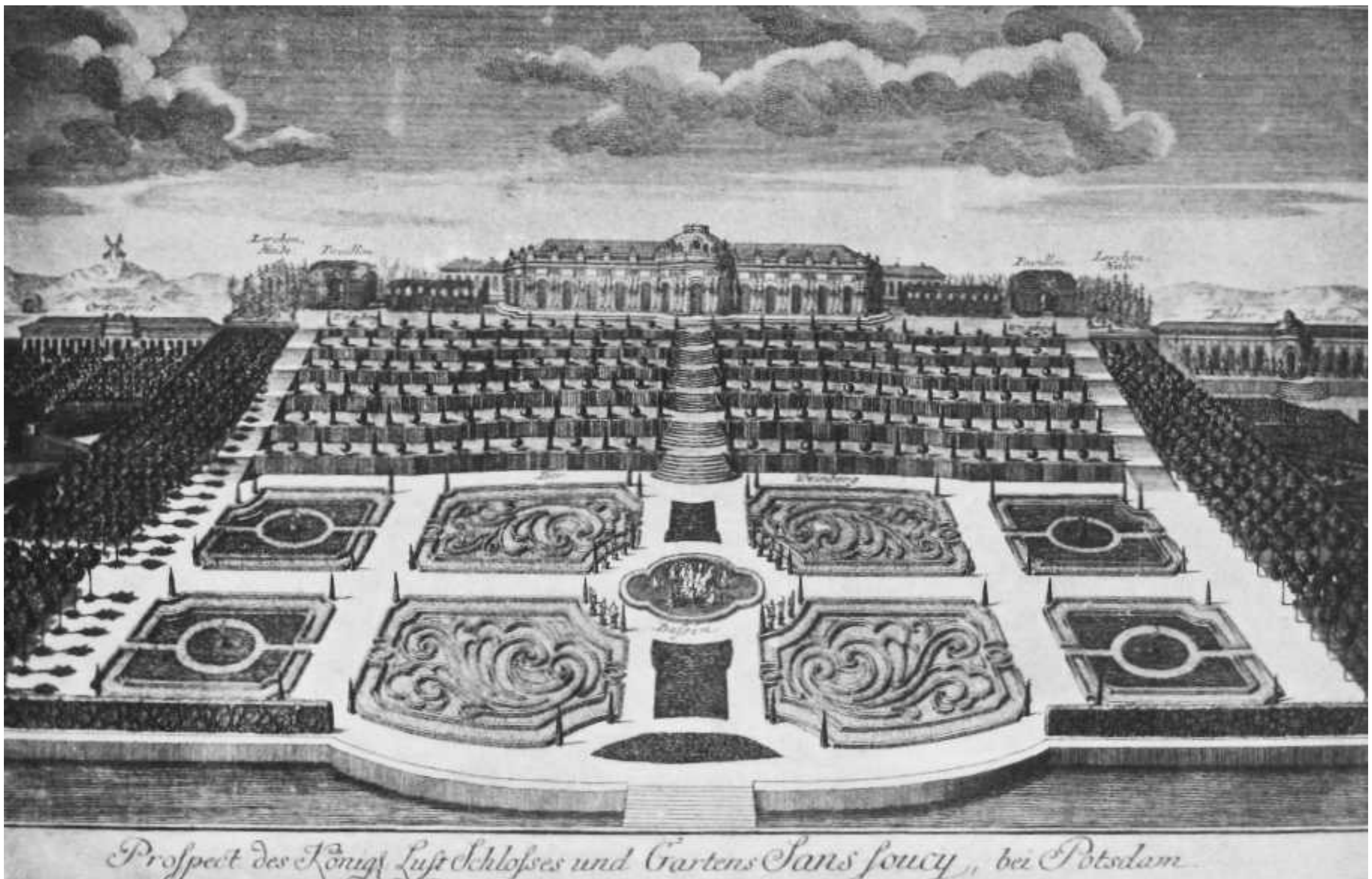
Euler und die Springbrunnen von Sanssouci

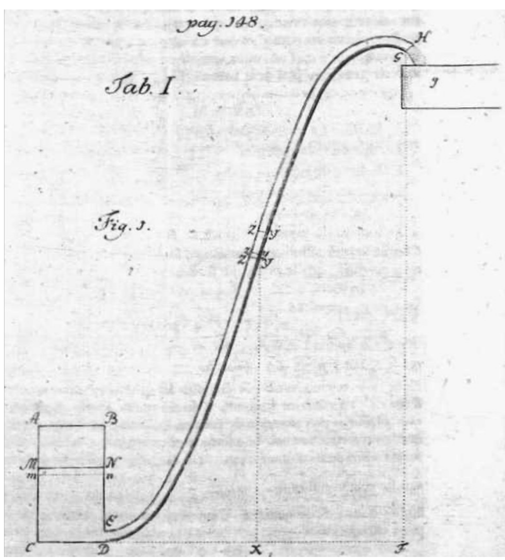
Ein Beispiel für das Auseinanderklaffen von Theorie und Praxis?

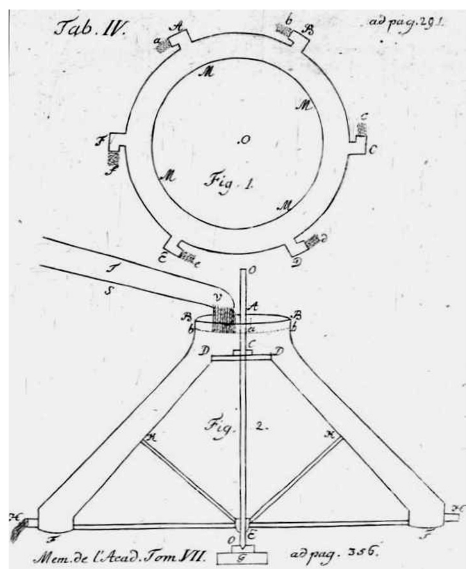
Michael Eckert, Deutsches Museum München

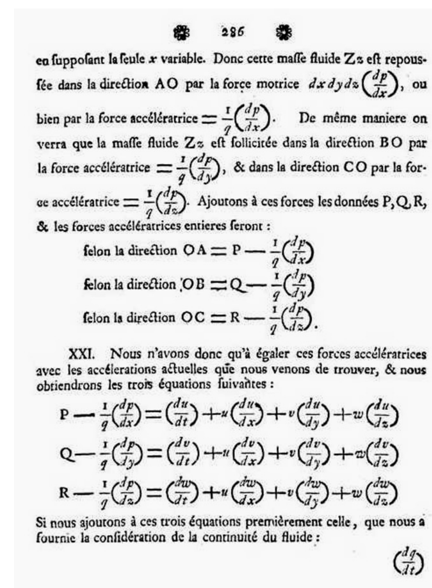


Euler – ein praktischer Akademiker

1741/1749	Scientia Navalis
1744/1745	Neue Grundsätze der Artillerie
1749/1752	**Sur le mouvement de l'eau par des tuyaux de conduite**
1750/1760	Recherches sur le mouvement des rivieres
1751/1752	Recherches sur l'effet d'un machine hydraulique
1755/1757	Principes generaux du mouvement des fluides







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en ſuppoſant la ſeule x variable. Donc cette maſſe fluide Zz eſt repouſſée dans la direction AO par la force motrice $dx\,dy\,dz\left(\frac{dp}{dx}\right)$, ou bien par la force accélératrice $= \frac{1}{q}\left(\frac{dp}{dx}\right)$. De même maniere on verra que la maſſe fluide Zz eſt ſollicitée dans la direction BO par la force accélératrice $= \frac{1}{q}\left(\frac{dp}{dy}\right)$, & dans la direction CO par la force accélératrice $= \frac{1}{q}\left(\frac{dp}{dz}\right)$. Ajoutons à ces forces les données P, Q, R, & les forces accélératrices entieres ſeront :

ſelon la direction OA $= P - \frac{1}{q}\left(\frac{dp}{dx}\right)$

ſelon la direction OB $= Q - \frac{1}{q}\left(\frac{dp}{dy}\right)$

ſelon la direction OC $= R - \frac{1}{q}\left(\frac{dp}{dz}\right)$.

XXI. Nous n'avons donc qu'à égaler ces forces accélératrices avec les accélerations actuelles que nous venons de trouver, & nous obtiendrons les trois équations ſuivantes :

$$P - \frac{1}{q}\left(\frac{dp}{dx}\right) = \left(\frac{du}{dt}\right) + u\left(\frac{du}{dx}\right) + v\left(\frac{du}{dy}\right) + w\left(\frac{du}{dz}\right)$$

$$Q - \frac{1}{q}\left(\frac{dp}{dy}\right) = \left(\frac{dv}{dt}\right) + u\left(\frac{dv}{dx}\right) + v\left(\frac{dv}{dy}\right) + w\left(\frac{dv}{dz}\right)$$

$$R - \frac{1}{q}\left(\frac{dp}{dz}\right) = \left(\frac{dw}{dt}\right) + u\left(\frac{dw}{dx}\right) + v\left(\frac{dw}{dy}\right) + w\left(\frac{dw}{dz}\right)$$

Si nous ajoutons à ces trois équations premièrement celle, que nous a fournie la conſidération de la continuité du fluide :

$$\left(\frac{dq}{dt}\right)$$

„... zweitklassig als Physiker...“

„M. Euler paraissait quelquefois ne s‘occuper que du plaisir de calculer... dans M. Euler, le métaphysicien, ou mème le physicien, n‘a pas été si grand que le géomètre... souvent il ne cherchait qu‘à montrer les forces et les ressources de son art...“ (Condorcet 1783)

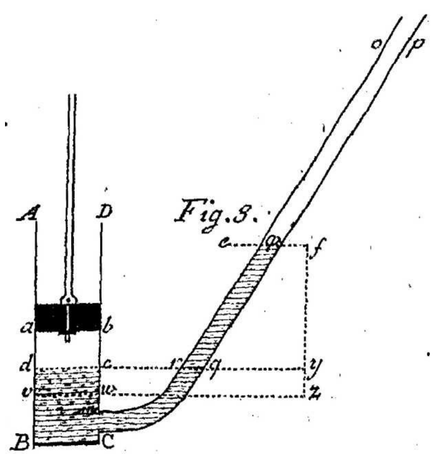
„The physical universe was an occasion for mathematics to Euler, scarcely a thing of much interest in itself; and if the universe failed to fit his analysis it was the universe which was in error.“ (E. T. Bell 1937)

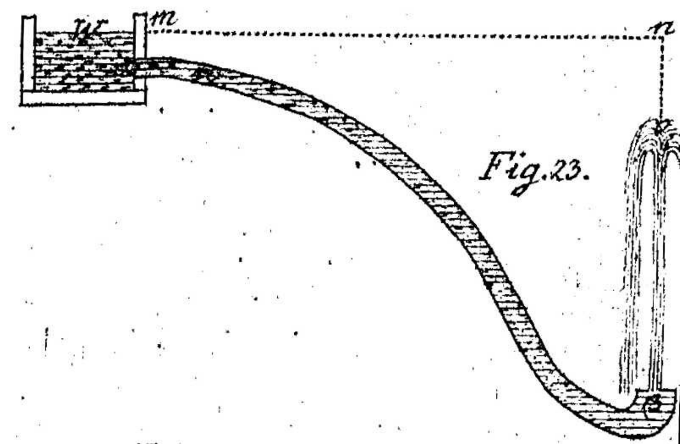
„Der geniale Mathematiker Euler war zweitklassig als Physiker ...“ (A. Hermann 1991)

“When Euler applied his equations to design a fountain for Frederick the Great of Prussia, it failed to work...Unfortunately, he omitted the effects of friction, with embarrassing practical consequences.” (S. Perkovitz 1999)

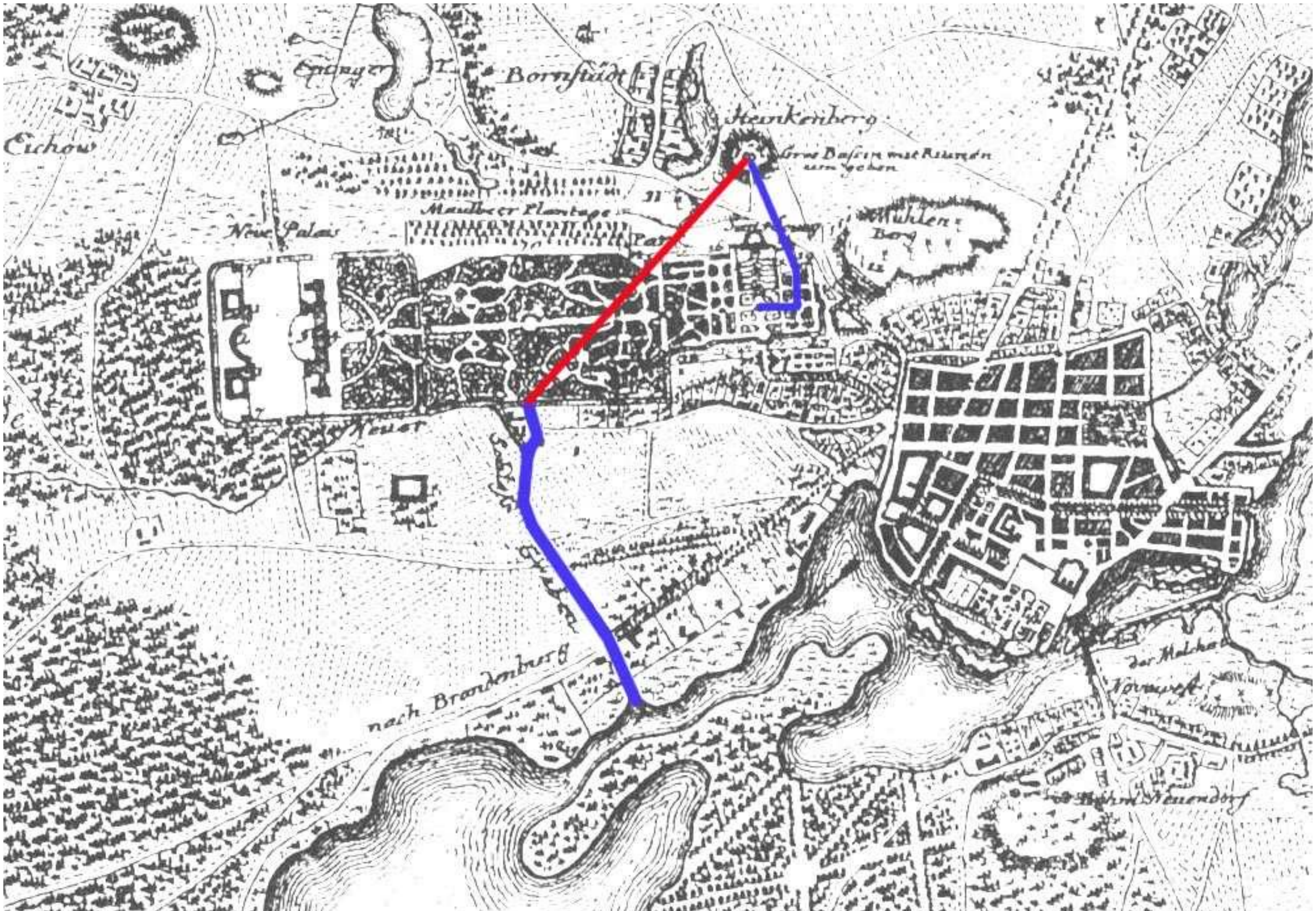
Der Spott des Königs

Je voulus faire un jet-d'eau en mon Jardin; le Ciclope Euler calcula l'éffort des roües, pour faire monter l'eau dans un bassin d'où elle devoit retomber par des Canaux, afin de jaillir à Sans-Souci. Mon Moulin a été éxécuté géométriquement, et il n'a pu élever une goute d'eau à Cinquante pas du Bassin. Vanité des Vanités; Vanité de la géométrie.” (Frederick II to Voltaire, 25 January 1778)





Wasserkunst in Sanssouci



Heinrich Ludewig Manger's
Königl. Preuß. Ober-Hof-Baurath und Garteninspectors

Baugeschichte
von
Potsdam,

besonders
unter der Regierung
König Friedrichs des Zweiten.

Erster Band,
welcher die Baugeschichte von den ältesten Zeiten
bis 1762 enthält.

Berlin und Stettin,
bei Friedrich Nicolai, 1789.

vier bleyerne Grouppen, sechsfüßiger Proportion; Giese goß überdieß die metallenen Cylinder, oder Stiefel zur Mühle und verschiedene Arbeiten von Bley, weil er im Gießen sehr geschickt war. 1748

Die hölzernen gebohrten Röhren mit den eisernen Schraubenringen waren nun endlich auch verlegt, erfuhren aber ebenfalls gar bald das Schicksal der vorigen aus verschiedenen Stäben zusammengesetzten, nämlich sie konnten den Druck des Wassers nicht aushalten und zersprangen. Dem Könige mußte also vorgestellet werden, daß auf keine Weise das Wasser anders herauf in den Sammelkasten zu bringen möglich wäre, als wenn solches vermittelst guter gegossenen eisernen, oder bleyernen Röhren geschähe, wozu denn dergleichen von Cassel und vom Harze vorgeschlagen wurden.

Es konnte nicht anders seyn, als daß der König wegen der durch zweierley hölzernen Röhren verwendeten, gleichsam weggeworfenen Summen, sehr unwillig ward, denn er verlangte durchaus und mit Recht, daß diejenigen, welche etwas angäben, vorher von dem Effekte durch Erfahrungen sicher seyn müßten. Er äußerte indessen den gerechten Unwillen nicht auf eine strenge Art, sondern da er von neuem zu eisernen Röhren Gelder assignirt hatte: so ließ er ein Paar Esel in Lebensgröße mit Oelfarbe auf Leinwand mahlen, mit Rähmen umgeben, und die Unterschrift beifügen:

Hollaandse Fontaenen-Maacker.

Diese recht schön, und gänzlich nach der Natur getroffenen Thiere, sollten mit Wasserfarbe, etwas anders vorstellend übermahlt, und an ein lischen Geheimdenrath Waitz empfohlener Fontainier oder Grottier, Nahmens J. F. George, der eigentlich ein Rothgießer, und als solcher ganz wohl zu gebrauchen war; einige Angaben zu Wasserwerken, oder zu Verbesserung schon vorhandener, durfte man aber von ihm allein ebenfalls nicht erwarten. 1748

Zweite Windmühle.

Da man überhaupt gar zu wohl wahrnahm, daß es mit Heraufschaffung des Wassers in den Sammelkasten auf den Höneberg vermittelst der einzigen angelegten Mühle sehr langsam zugehen würde, wenn auch die nunmehr verlegten eisernen und bleyernen Steigeröhren allen Druck gehörig aushielten: so wurde im Jahre 1754 noch eine der vorigen völlig gleiche Mühle mit sechs metallenen Stiefeln in einem Grunde zwischen Bornstädt und dem Höneberge gebauet, welche der untern zu Hülfe aus ihren Brunnen eben so viel Wasser, als jene, herauf pressen sollte.

Erster Wasserstrahl in Sans-Souci.

Indessen hatte die erstere doch nach und nach so vieles Wasser herauf geleyert, (mich deuchtet der Ausdruck ist passend genug, denn es ging damit erbärmlich langsam zu) und vieler gefallener Schnee und Regen hatten zugleich so gute Dienste geleistet, daß der Kessel beynahe halb voll geworden war. Man verkündigte also dem Könige, daß nunmehro mit dem Sprunge eines Wasserstrahls von ansehnlicher Höhe in dem Becken vor der Grotte eine Probe gegeben werden könnte. Der König setzte dazu den folgenden Tag an, welches zwar der stille Freitag dieses 1754sten Jahres, demungeachtet aber ein sehr stürmischer Tag war, und Er hatte für alle zeither angewandte Kosten das Vergnü-

1748 gen, diesen Strahl beinahe eine Stunde lang springen zu sehen, der vielleicht 50 Fuß Höhe würde erreicht haben, wenn es der Wind zugelassen hätte. Sodann war das Wasser alle, und nach dem Vorhergehenden zu urtheilen, war es wahrscheinlich, daß übers Jahr wieder so viel da seyn würde, um die Probe wiederholen zu können.

Neue Epoche der Wasserwerke in S. S. unter Pfannenstiehl.

Allein so lange sollte es doch damit von Rechtswegen nicht dauern; denn kurz darauf erschien derjenige, welcher allen diesen Anstalten zu Wasserwerken die lezte Hülfe geben wollte, nachdem alle vorherigen Wasserkunstdirectörer und Wasserkünstler ihren gnädigen Abschied erhalten hatten. Es war Johann Valentin Pfannenstiehl, ehemaliger Kupferschmidt und Spritzenbauer, nachheriger (seiner Sage nach) Stücklieutenant Sr. Churfürstlichen Gnaden zu Mainz. Er meldete sich mit seiner Kunst immediate, und versprach mehr, als sich hoffen ließ. Baumann hatte sich von Potsdam entfernen und nach Berlin begeben müssen. Hildebrant und Büring lehnten die Aufträge dieser Art möglichst von sich ab: Fredersdorf bekam also Befehl, von dem sich gemeldeten neuen Fontainier Pfannenstiehl Plane und Anschläge von der Ausführung zu fordern, und das Fernere mit ihm zu verabreden. Er übergab solche, und der Verfasser hatte die Ehre, daß sie ihm gezeigt wurden, ungeachtet sie nur zur Einsicht des Königs und Fredersdorfs bestimmt waren. Da der Plan schon höchst genehmigt war, forderte man sein Gutachten nicht und Pfannenstiehl mußte also schweigen.

Eulers Auftrag

« ... j'ai l'honneur de Vous marquer que j'expédiai hier au Roy mes recherches sur la lotterie projettée, et que j'espère de venir à bout en quelques jours de celles sur la machine hydraulique ... » (Euler an Maupertuis, 18. September 1749)

« Je prend la liberté de vous addresser mes recherches sur la Machine Hydraulique de Sans Soucy ... je crains fort qu'il s'en faudra beaucoup qu'elle monte à la hauteur que Le Roy souhaite... » (Euler an Maupertuis, 21. September 1749)

« Comme Sa Majesté le Roy de Prusse, Notre très gracieux Souverain, a reçu les calculs que le professeur Euler Lui a adressé au sujet de la Machine de Sans-Souci et qu'Elle en est fort contente, Sa Majesté veut bien lui témoigner tout le gré ... » (Friedrich II. an Euler, 27. September 1749)

« ... en cas que l'expérience de Mariotte ne fût pas juste, ou gâtée par une faute d'impression, je ne saurois rien déterminer sur l'épaisseur des tuyaux dans le cas dont il s'agit, à moins qu'on ne fît de nouveau des expériences sur la force que des tuyaux de plomb sont capables de soutenir. Car on risqueroit trop si l'on vouloit confier au seul hazard la détermination de l'épaisseur des tuyaux ... » (Euler an Maupertuis, 30. September 1749)

Eulers Warnung

« Car sur le pied qu'elles se trouvent actuellement, il est bien certain, qu'on n'éleveroit jamais une goutte d'eau jusqu'au réservoir, et toute la force ne seroit employée qu'à la destruction de la machine et des tuyaux. » (Euler to Frederick, 17 October 1749)

« La véritable cause de ce fâcheux accident consistoit uniquement en ce que la capacité des pompes étoit trop grande, et à moins qu'on ne la diminue très considérablement, ou en diminuant leur diamètre ou leur hauteur, ou le nombre des jeux qui repond à un tour de moulin, la machine ne sera pas en état de fournir une seule goûte d'eau dans le réservoir. » (Euler to Maupertuis, 21 October 1749)

« J'ai reçu votre lettre du 17^{e} de ce mois, contenant les remarques, que vous avez fait sur vos calculs sur les pompes et les tuyaux de la Machine de Sans-Souci. Elles M'ont été fort agréables, et Je vous suis bien obligé de la peine que vous en avez pris. » (Frederick to Euler, 21 October 1749)

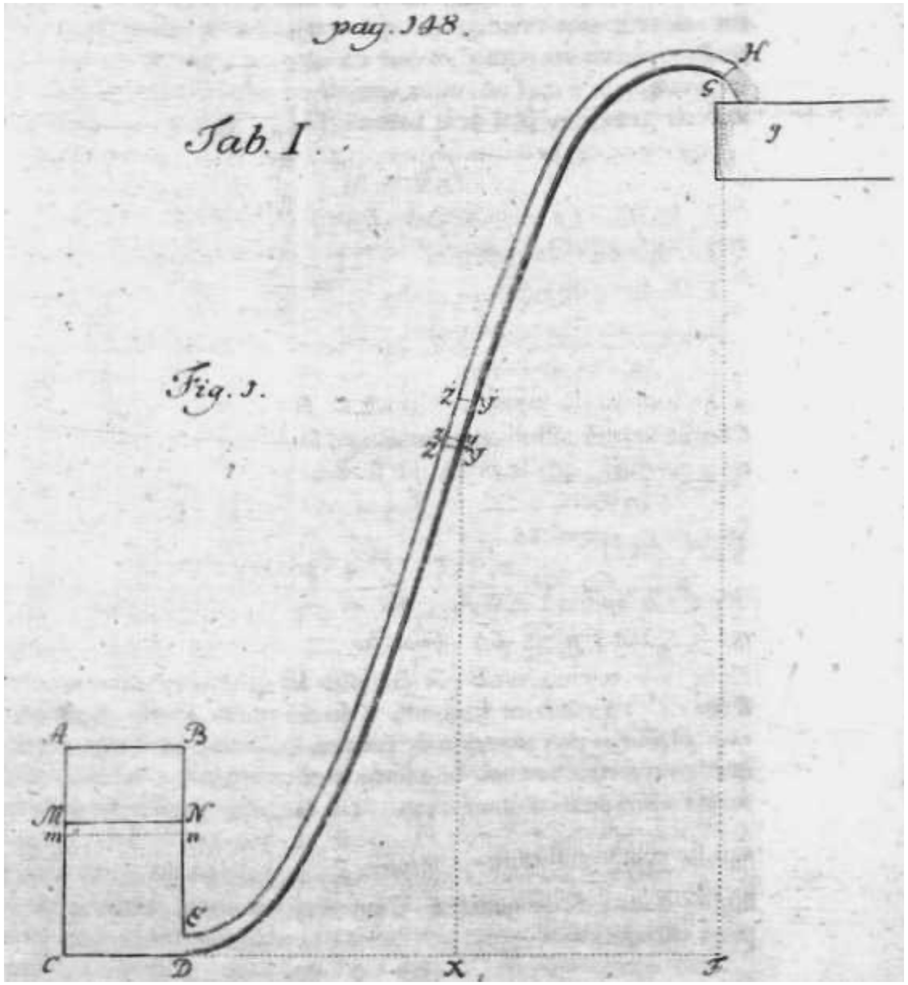
Eulers hydraulische Akademieberichte

Sur le mouvement de l'eau par des tuyaux de conduite (E 206)
(vorgetragen am 23. Oktober 1749; veröffentlicht in *Mémoires de l'académie des sciences de Berlin* 8, (1752) 1754, S. 111-148.

Discussion plus particuliere de diverses manieres d'elever de l'eau par le moyen des pompes avec le plus grand avantage (E 207)
(vorgetragen am 20. November 1749; veröffentlicht in *Mémoires de l'académie des sciences de Berlin* 8, (1752) 1754, S. 149-184.)

Maximes pour arranger le plus avantageusement les machines destinées è élever de l'eau par le moyen des pompes (E 208)
(vorgetragen am 5. Februar 1750; veröffentlicht in *Mémoires de l'académie des sciences de Berlin* 8, (1752) 1754, S. 185-232.)

Eulers Theorie der Rohrströmung

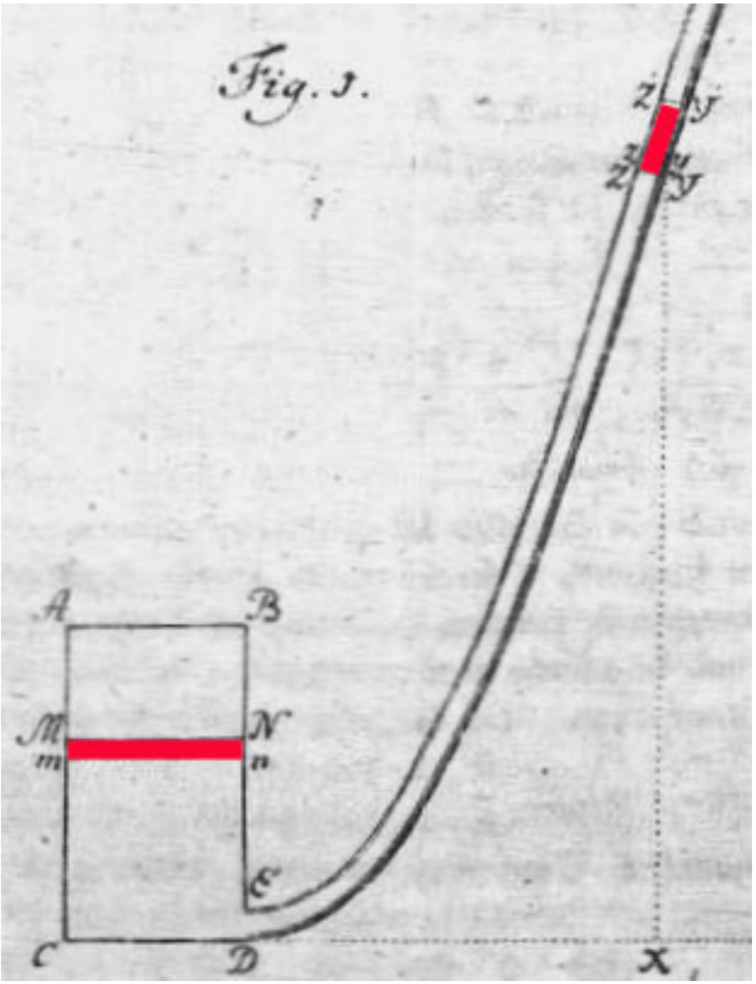


1. Bewegungsgleichung aufstellen

2. Differentialgleichung lösen

3. Anwendungen erklären

Bewegungsgleichung



Kraft = Masse x Beschleunigung

- Beschleunigung von Y nach *Y'*, die der Kolbenbewegung *Mm* entspricht?

- Kräftegleichgewicht für die Masse *Yzzy?*

De ces deux forces résultera donc une force motrice, qui pousse la couche Y Z zy en arrière, & qui sera $= \frac{1}{4}\pi zz dp$: à laquelle ajoutant la force, qui résulte du poids de la couche qui étoit $= \frac{1}{4}\pi zz dy$, cette couche sera conjointement poussée en arrière par la force motrice $\frac{1}{4}\pi zz\,(dp + dy)$, qui étant divisée par la masse même de la couche $\frac{1}{4}\pi zz\,ds$, donne la force accélératrice $= \frac{dp + dy}{ds}$, qui étant contraire à la direction YY', il faut égaler V à $- \frac{dp - dy}{ds}$.

De là nous obtiendrons cette équation

$$dp + dy = - \frac{aa\,ds}{zz} \cdot \frac{dv}{dr} + \frac{4a^4 S\,ds}{z^5} \cdot v$$

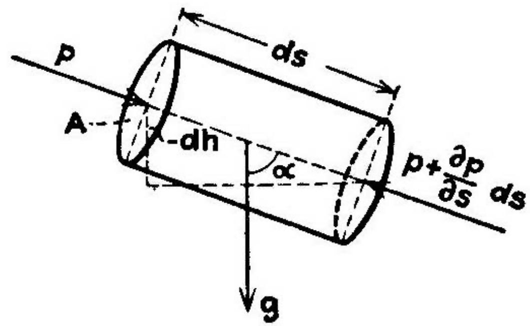
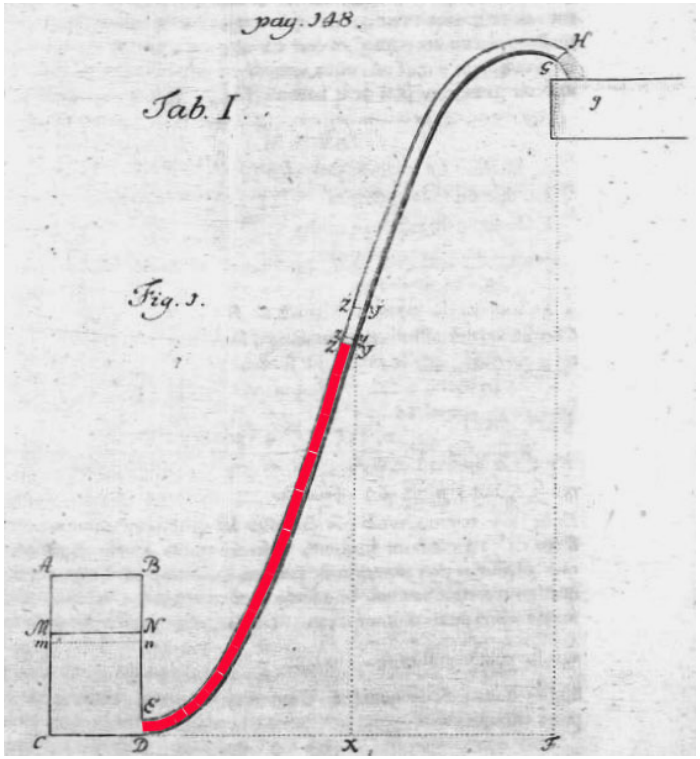


FIG. 2.—Forces on an element of ideal fluid.

$$\underbrace{\rho dA ds \cdot \frac{Dw}{dt}}_{\text{mass} \times \text{acceleration}} = \underbrace{\rho g dA ds \cos\alpha}_{\text{gravity force}} + \underbrace{dA\left\{p - \left(p + \frac{\partial p}{\partial s}ds\right)\right\}}_{\text{pressure force}}$$

$$\frac{\partial p}{\partial s} + g\rho\frac{\partial y}{\partial s} = -\rho\left(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial s}\right)$$

Integration



Lösung der partiellen Dgl.

$$\frac{\partial p}{\partial s} + g\rho \frac{\partial y}{\partial s} = -\rho\left(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial s}\right)$$

ergibt

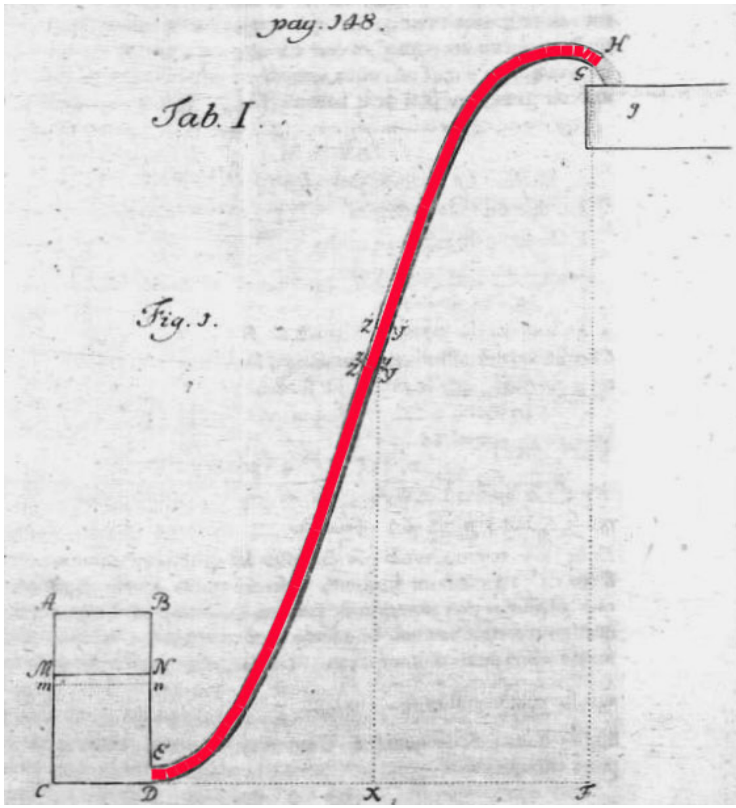
$$p = p_0 + g\rho(y_0 - y) + \frac{\rho}{2}(w_0^2 - w_y^2) - \rho\int_0^Y \frac{\partial w}{\partial t}\,ds$$

(nichtstationäre „Bernoulli Equation“)

Bestimme Integrationskonstanten bei *CD*:

$$p = (k + b - r - y)g\rho + \frac{\rho}{2}(w_p^2 - w^2) - \rho\frac{dw_p}{dt}(b - r) - \rho\int_D^Y \frac{\partial w}{\partial t}\,ds$$

$$p=(k+b-r-y)g\rho+\frac{\rho}{2}(w_p^2-w^2)-\rho\frac{dw_p}{dt}(b-r)-\rho\int_D^Y\frac{\partial w}{\partial t}ds$$



$p = 0$ bei GH ergibt Dgl. für die Kolbenbewegung; deren Lösung ergibt:

$$r=\frac{L}{\frac{a^4}{c^4}-1}\ln\frac{k-h}{k-h-\frac{w_p^2}{2g}(\frac{a^4}{c^4}-1)}$$

Im Grenzfall $k-H>>\frac{w_p^2}{2g}(\frac{a^4}{c^4}-1)$:

$$w_p=\sqrt{\frac{2g(k-h)}{L}r}\ .$$

$$p(y)=g\rho\{k-y+b-r+\frac{k-h}{L}(1-\frac{a^4}{z^4}-b+r)-\frac{(k-H)}{L}a^2\int\frac{ds}{z^2}\}$$

$$\approx g\rho\{k-y-\frac{(k-H)}{L}a^2\int\frac{ds}{z^2}\}$$

Weitere Probleme

PROBLEME III.

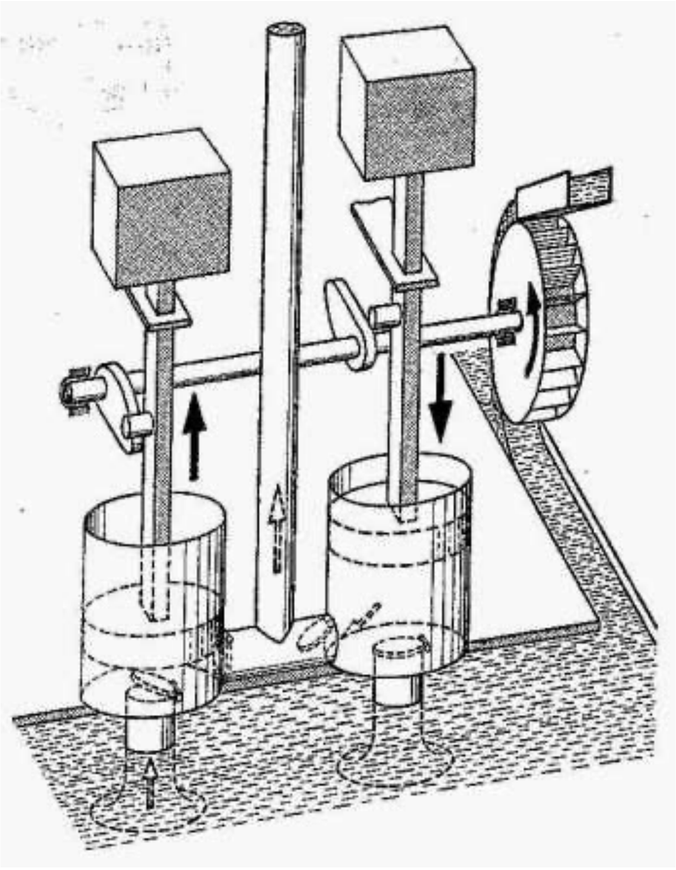
XIX. *La hauteur du réſervoir* FG = *g étant extrêmement grande par rapport à la hauteur de la pompe, déterminer le tems, pendant lequel le piſton eſt pouſſé en bas depuis* AB *presqu'en* E, *lorsqu'il eſt pouſſé par la force conſtante* = K.

PROBLEME IV.

XX. *Si deux pompes égales, dont les piſtons ſont auſſi pouſſés par des forces égales, agiſſent alternativement, de ſorte que pendant qu'une aſpire l'autre refoule, & que toutes les deux refoulent l'eau dans le même tuyau montant* DEGH, *pour la dégorger dans le réſervoir* I *élevé à une hauteur fort grande, déterminer le tems du jeu des piſtons, la preſſion que le tuyau a à ſoutenir, & la quantité d'eau, qui ſera fournie dans le réſervoir pendant une heure.*

PROBLEME V.

XXXIV. *Le tems du jeu des piſtons étant donné, avec les meſures de toutes les parties de la machine deſtinée à élever de l'eau, trouver la force qui doit agir ſur les piſtons, & la preſſion, que le tuyau montant aura à ſoutenir en bas.*



qui ont entrepris la conſtruction d'une telle machine ; puiſque les tuyaux ne manqueront pas de créver, quoiqu'on ait crû avoir pris toutes les précautions pour prévenir cet accident facheux. Je rapporterai un exemple, d'où l'on verra combien la preſſion ſur le tuyau peut devenir grande au delà de la hauteur ſimple de l'eau dans le tuyau.

EXEMPLE.

XLIII. La machine propoſée avoit ces meſures.

Le diametre des pompes	=	$\frac{4}{3}$	pieds	= a
La hauteur du jeu des piſtons	=	4.	pieds	= b
Le diametre du tuyau montant	=	$\frac{3}{4}$	pieds	= c
La longueur du tuyau	=	3000	pieds	= l
La hauteur du tuyau	=	60	pieds	= g.

Chaque jeu des piſtons s'achevoit en 6 ſecondes. Cela poſé, on demande la preſſion, que le tuyau dût ſoutenir en bas.

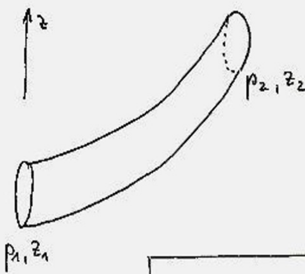
Ayant donc $t = 6''$ & poſant cette preſſion équivalente à la hauteur p, on aura :

$$p = 60 + \frac{0{,}256 \cdot \frac{16}{9} \cdot 4 \cdot 3000}{\frac{9}{16} \cdot 36} \text{ pieds.}$$

qui ſe réduit à $p = 60 + 270 = 330$ pieds.

Donc, ſi le tuyau n'avoit pas été aſſez fort pour porter une colomne d'eau de 330 pieds de hauteur, il ſeroit crévé infailliblement ; quoique la hauteur de l'élevation de l'eau ne fût que de 60 pieds, de ſorte que le tuyau dût ſoutenir une force plus de 5 fois plus grande, que le ſimple poids de la colomne d'eau. De là on connoîtra la force, qui agit ſur chaque piſton, qui étoit $= \frac{\pi}{4} \cdot \frac{16}{9} \cdot 330 = 461$ pieds cubiques d'eau, & la quantité d'eau élevée dans une heure $= 6701$ pieds cubiques.

DIS-



$$\rho F ds \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s}\right) = -dp\, F - \rho F ds\, g \frac{dz}{ds} - \tau 2\pi r\, ds$$

Annahme: konstanter Rohrquerschnitt $\rightarrow \frac{\partial u}{\partial s} = 0$

$$\rho \frac{du}{dt} l = p_1 - p_2 + \rho g (z_1 - z_2) - \frac{2\tau l}{r}$$

$$\tau = c_f \cdot \frac{\rho u^2}{2} \qquad c_f = \frac{f}{4}$$

$$p_1 = p_2 + \rho g h + \rho \frac{du}{dt} l + \rho f \cdot \frac{u^2}{4r} l$$

Außendruck = 0

Darcy-Weisbach-Koeff. ~ 0.01 → 0.06 abh. von R und Rauhigkeit der Rohrwand

$$\frac{p_1}{\rho g} = h + l \left[\frac{du/dt}{g} + f \cdot \frac{u^2}{4rg}\right]$$

Zahlenbeispiel: $\frac{du}{dt} \approx 0.9 \frac{m}{s^2}$; $u \approx 2.7 \frac{m}{s}$; $r \approx 0.1 m$

→ Druckhöhe $= h + l\, [0.09 + f \cdot 1.5]$ (60) (3000)

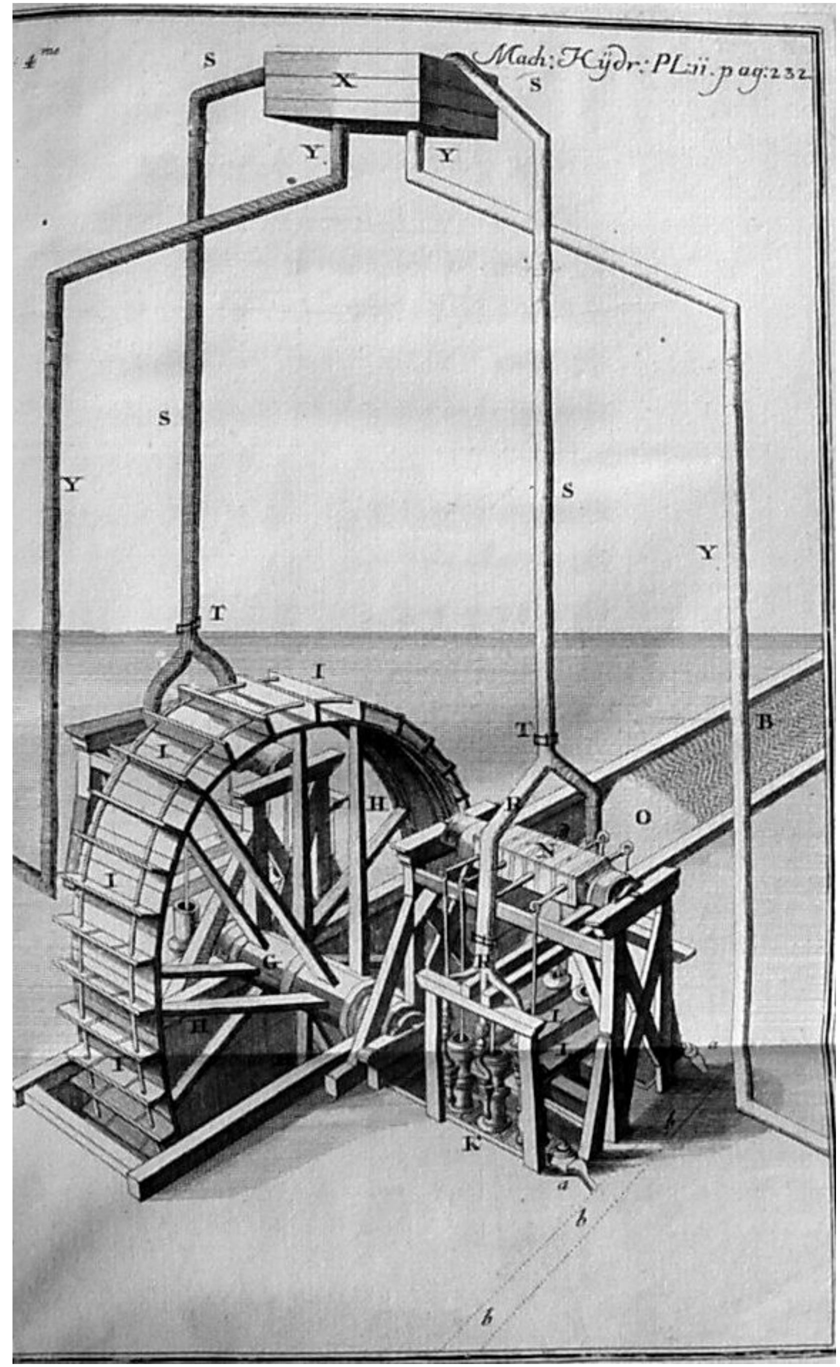
0.015 für glatte Rohrwand
0.091 für rauhe "

$= 60$ Fuß $+ \underbrace{3000 \cdot 0.09}_{270 \text{ Fuß}} + \underbrace{3000 \cdot 0.015}_{45 \text{ Fuß}}$

$\underbrace{3000 \cdot 0.091}_{273 \text{ Fuß}}$

Pour que la meme force qui agit sur les pistons des pompes soit en état de fournir dans le réservoir la plus grande quantité d’eau, il faut avoir soin de faire le tuyau montant aussi large qu’il sera possible (...)

Pour fournir une plus grande quantité d’eau dans le réservoir par la meme force qui agit sur les pistons, il faut rendre le tuyau montant aussi court qu’il sera possible.



Fazit

- Eulers Analyse des Sanssouci-Problems war korrekt. Sie begründete die moderne Hydraulik. Er stellte die Bewegungsgleichung für das Problem auf (die eindimensionale „Euler-Gleichung“ für einen Stromfaden) und leitete daraus die nicht-stationäre „Bernoulli-Gleichung“ ab.
- Er leitet aus seiner Lösung praktische Regeln ab, doch sie wurden ignoriert. Das Wasserkunst-Projekt in Sanssouci scheiterte nicht, weil Eulers Theorie nicht praxisgerecht war, sondern weil der König unfähige Praktiker pfuschen ließ und vor den hohen Ausgaben für ein so aufwendiges Projekt zurückschreckte.
- Als Euler kurze Zeit später die allgemeinen Bewegungsgleichungen für reibungsfreie Fluide formulierte, beruhte dies auf jahrelanger Erfahrung mit praktischen Problem wie dem in Sanssouci. Auch wenn es paradox erscheint: Die Theorie idealer Fluide wurzelte in praktischen Strömungsproblemen.

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Euler and the Fountains of Sanssouci

MICHAEL ECKERT

Communicated by M. FOLKERTS and R. STUEWER

Leonhard Euler (1707–1783) is known as a mathematical genius. He is also praised as a pioneer in theoretical mechanics. The equations that describe the motion of a top are called "Euler's equations." Another set of "Euler's equations" is used in hydrodynamics to account for the flow of ideal fluids. At the same time, Euler is said to have failed when dealing with a practical hydrodynamical problem: The King of Prussia, Frederick the Great, engaged Euler to calculate the hydraulics for a fountain in his Park at Sanssouci in Potsdam, but the project was never completed because Euler's theory seemed useless for this task. This failure at Sanssouci became a symbol for the schism between theory and practice in the 18th century and beyond.

Euler's alleged failure at Sanssouci has never been scrutinized by historians of science and technology – despite the frequent references to it in comments on the gap between hydrodynamics and hydraulics, ideal and real fluids, and other forms of the devide between theory and practice. One physicist cited the case of the "mathematician Euler" as illustrative of the difference between someone dealing with ideal and real fluids: "Unfortunately, he omitted the effects of friction, with embarrassing practical consequences. When Euler applied his equations to design a fountain for Frederick the Great of Prussia, it failed to work."[1] One historian of physics argued, that Euler's failure can be traced to his incomplete understanding of the principle of conservation of energy: "The mathematical genius Euler was a second-rate physicist... He would have found the required information in the writings of Leibniz whom he revered."[2] A historian of technology cited Euler's failure as an example of futile attempts to apply mathematics to practical problems in the Age of Enlightenment: "Frederick the Great ordered in 1749 that Euler should calculate a machine to raise water, which should be used for the fountains at Sanssouci. But Euler's theory was not applicable for practical ends. The King recalled this 'according to scientific points of view' water raising construction still in

[1] Sidney Perkovitz, "The Rarest Element," *The Sciences*, *39* (January/February 1999), 34–38, here p. 38.

[2] Armin Hermann, *Weltreich der Physik. Von Galilei bis Heisenberg*, fifth edition (Stuttgart: Deutscher Taschenbuch Verlag, 1991), pp 80-81: „Der geniale Mathematiker Euler war zweitklassig als Physiker... Aufschluß gefunden hätte er in den Schriften des von ihm verehrten Leibniz."

1778 in a letter to Voltaire and mocked: 'Il n'a pu élever une goutte d'eau à cinquante pas du bassin. Vanité des vanités! Vanité de la géométrie!'"[3]

But what is the evidence for Euler's alleged failure? What was the nature of the hydrodynamical problem, and how was it related to the practical problem of constructing a fountain? What was the contemporary 18th-century knowledge of hydrodynamics and hydraulics upon which Euler based his work at Sanssouci?

Bungling at Sanssouci

Before I attempt to provide answers to these questions, we must examine the history of the fountain at Sanssouci.[4] The project started in 1748, a year after the inauguration of the newly erected castle. The King gave an order to his architect, Johann Baumann, to design a water system in his Park with several fountains, to be fed by the Havel river some distance away. In particular, he wished to have a major fountain whose jet would rise to a height of at least 100 feet. Baumann had come to the court of the Prussian King from the Netherlands, a country known for its canals and water constructions, but he had no experience with fountains and the hydraulics required for their operation: he thus hired a garden technician from Amsterdam, Heintze, who previously had built minor fountains in smaller gardens, but he, too, had no expert knowledge about projects on the scale that the King had in mind.

A first plan involved the use of a steam engine as the driving force for the fountain jet, but the method for using such a machine was still unexplored. Another plan involved the construction of an extended system of canals with water wheels to raise the water to an elevated reservoir, from where it then could drive the fountain, but the high cost of constructing the canals prevented the realization of this plan. Another alternative was to use wind power: Pumps, driven by a windmill, should raise the water of the Havel river to an elevated reservoir. This proposal was executed; it involved the construction of a water reservoir on top of a hill 150 feet above the river level, with a windmill-driven

[3] Friedrich Klemm, *Zur Kulturgeschichte der Technik. Aufsätze und Vorträge 1954–1978*, 2nd edition (München: Deutsches Museum, 1982), p. 201: „Friedrich der Große ließ 1749 von Euler eine Wasserhebemaschine berechnen, die die Wasserkünste in Sanssouci betreiben sollte. Aber Eulers Berechnungen waren praktisch nicht zu verwerten. Der König erinnerte sich noch 1778 in einem Brief an Voltaire dieses, nach wissenschaftlichen Gesichtspunkten' ausgeführten Wasserhebewerkes und spottete:‚Il n'a pu élever une goutte d'eau à cinquante pas du bassin. Vanité des vanités! Vanité de la géométrie!'"

[4] The following is based on a description due to one of the King's architects: Heinrich Ludewig Manger, *Baugeschichte von Potsdam*, 3 Vols. (Berlin: Friedrich Nicolai, 1789), here Vol. 1, pp. 91–106 and Vol. 2, p. 270. Later accounts, based on this source, are H. E. R. Belani, *Geschichte und Beschreibung der Fontainen-Anlagen in Sanssouci* (Potsdam: Otto Jahnke, 1843); M. Gottgetreu, "Der Fontainenbau in Sanssouci," *Zeitschrift für Bauwesen, 2*, (1852), 251–256, and Paul Artelt, *Die Wasserkünste von Sans-souci. Eine geschichtliche Entwickelung von der Zeit Friedrichs des Großen bis zur Gegenwart* (Berlin: Schwarz, 1893). According to the information of the Geheimes Staatsarchiv Preussischer Kulturbesitz, Berlin, archival sources concerning the construction of the fountains at Sanssouci were destroyed in World War II.

water pump half-way between the river and the reservoir. The water had to be guided by a canal from the river to the site of the pump station, from where it would be pumped through pipes up the hill and into the reservoir. Other pipes would connect the reservoir with the main fountain and four smaller fountains. Furthermore, water from the reservoir would be guided into a grotto and over cascading waterfalls. Altogether 62 destinations were foreseen for the water from the reservoir.

Construction began in the summer of 1748. The canal from the Havel river to the pump station was finished by November. The windmill and the pumps were finished by the end of the year. The mechanism used to transmit the motion of the windmill to the pumps was described as clumsy, but it seemed to work. The pumps also were connected to a mechanism that could be set into motion by horses (Göpelwerk) if there were no wind.

So far the project progressed according to expectations. But problems arose when the pump station was connected to the elevated reservoir. Eight hundred tree trunks of spruce, each 24 feet long, were cut into boards that were assembled like wooden barrels, with iron bands around them to withstand the pressure of the water. The wooden pipes between the pumps and the resrvoir had an inner diameter of 7 to 9 inches, those from the reservoir to the fountains 12 to 16 inches. By the spring of 1749, the pipes were ready for a first test: Water was pumped into the pipeline, but it reached only about halfway up to the reservoir before the pipes near the pumps at the lower end began to burst.

After this failure the tubes were replaced by spruce tree trunks whose cores had been drilled out with an inner diameter of 3 to 5 inches. Between March and December of 1749, 8000 feet of such tubes were assembled. To protect them against bursting, iron bands were fitted around them; furthermore, five copper tanks (Windkessel) with a total weight of about 1.5 tons were placed along the pipeline to protect the tubes against sudden pressure changes. But the result was the same: the lower tubes again burst on the first trials. The King was upset and reproached Baumann and Heintze for not undertaking experiments on a smaller scale before final construction. He reportedly expressed his contempt for Baumann and Heintze by having oil paintings of donkeys entitled "Hollaandse Fonteinen maakers," hidden behind water-color paintings of landscapes. He planned to place these paintings on the houses of the unlucky fountain makers; the landscapes would vanish with the first rainfall and expose the derision underneath them. Although this plan was not executed, the King's intent was clear and drove Heintze to despair; he reportedly died of grief.

Heintze's successor as "fontainier" in the Park of Sanssouci was a man named Osten or van Osten. His term lasted from July 1752 to September 1753, when he was dismissed because he, too, was inexperienced in hydraulics. Osten, however, had replaced the hollow spruce tree trunks with metal tubes, which after the failure of the wooden tubes Baumann had ordered to be made of iron and lead in two different sizes, with inner diameters of 9 and 4 inches, for different sections of the pipeline, but only lead tubes of the smaller diameter were delivered. These now were used throughout. In October 1753, Osten was succeeded by yet another fountain maker, J. K. George, who is described as a good worker with some experience with fire hoses, but otherwise was as inexperienced in hydraulics as his predecessors. During his term in office a second windmill was constructed at a different site to raise water to the reservoir independently of the first one, but it never seems to have worked properly.

By the spring of 1754, an abundance of snow and rain together with the water-raising machine (described as "miserably slow") produced some tangible results. On Good Friday 1754, with a half-filled reservoir, the King was given a demonstration. But that day it was windy and the main fountain rose to only about half the height that the King had expected – and after an hour the reservoir was empty. A new fountain maker, Johann Valentin Pfannenstiel, was employed. He is described as an adventurer and presented plans for improving the machinery that revealed a total ignorance of the laws of hydraulics. For example, he planned to construct the pipeline to the reservoir so that the water would first descend for some distance to gain energy for its ascent. When his proposals were sent to one of the King's consultants (Privy Councellor von Waitz) and met with criticism, Pfannenstiel began a polemic that succeeded in persuading the King to allow him to continue his work without further external interference.

After another two years of unsuccessful bungling, the Seven Years' War broke out and the Sanssouci project was interrupted. When the war was ended in 1763, the King ordered new efforts to be undertaken to construct his fountain. When his consultant von Waitz presented him with an estimate of the cost, however, he was unwilling to spend the money. The basins in the Park and the reservoir on top of the hill had been finished, but those parts of the water-raising machinery that were not rotten were sold or used for other purposes. Only in 1841, under the reign of another King and almost a century after the initial plans for the fountain project had been presented, it was started anew – and brought to completion in only two years, using the power of steam engines and cast-iron pipelines of sufficient strength and appropriate dimensions (inner diameter 10 inches).

The report of the construction of the fountains at Sanssouci under Frederick the Great is pedantic in presenting its details and the names of those who had participated earlier in this effort: Baumann, Heintze, Osten, Pfannenstiel, George, and even the name of the King's consultant, von Waitz. Nowhere, however, do we find the name of Euler. Euler's role in the project evidently was quite different and unlike that suggested above in the introduction.

Euler's involvement

When Frederick the Great acceded to the throne in 1740, new traits of absolutist reign came to the fore in Prussia. While the military focus of his predecessor (known as the "Soldiers' King") was not abandoned as Prussia's first priority, Berlin now also was to become an "Athens" on the river Spree, a capital of art and science.[5] Among his first activities was the re-foundation of the Berlin Academy of Sciences. Established in 1700, the Academy had not lived up to the renown of its founder, Gottfried Wilhelm Leibniz. Among other celebrities in science and philosophy, Frederick the Great invited Leonhard Euler, then a member of the St. Petersburg Academy, to Berlin. Euler accepted the call and started to work for the new Berlin Academy immediately after his arrival in the summer of 1741. His numerous scientific papers were read at the Academy's regular

[5] H. Laitko, et al., *Wissenschaft in Berlin. Von den Anfängen bis zum Neubeginn nach 1945* (Berlin: Dietz Verlag, 1987), p. 74.

meetings and published in its proceedings. Furthermore, Euler was keen to demonstrate that he was a true believer in Leibniz's motto for the Academy, "theoria cum praxi." In April 1745, for example, he dedicated to the King a treatise on ballistics, Benjamin Robins's *New Principles of Gunnery*, which he had translated into German and augmented with many comments.[6] Robins's book, according to C. A. Truesdell, originally was merely a "little budget of rules, experiments, and guesses," but Euler's annotations transformed it into "the first scientific work on gunnery."[7]

Among the practical problems that Euler dealt with were such diverse tasks as the preparation of maps for a school atlas, the construction of canals, and the improvement of lenses.[8] Not surprisingly, therefore, Euler also became involved in the Sanssouci fountain project. There is no evidence that Euler became involved earlier than 1749, when the first bungling efforts with barrel-like tubes failed; perhaps this failure prompted his involvment. However, when Euler was asked for his advice, he was more than willing to contribute expertise in hydrodynamics to make the fountain project a success.

Euler's involvement, apparently, began late in the summer of 1749, when a new effort was made in the Park of Sanssouci to improve the water-raising machinery and the tubes for the pipeline to the elevated reservoir. On 21 September 1749 Euler sent a letter to the President of the Academy, Pierre Louis Moreau de Maupertuis (1698–1759):

> Je prend la liberté de vous addresser mes recherches sur la Machine Hydraulique de Sans Soucy, qui seront, à ce que je crois, suffisantes de diriger un tel ouvrage, desorte qu'on puisse etre ausseuré ne manquera pas de reussir. Je Vous prie, Monsieur, d'avoir la bonté de presenter ce paquet au Roy, si Vous n'y trouves rien qui le pourroit empecher. . .[9]

A few days later the King confirmed the receipt of Euler's calculations.[10] This was not the end of Euler's involvement. On 30 September 1749 he explained to Maupertuis « que mes calculs numeriques étoient fondés sur les mesures que Mr. Bauman s'étoit proposé de donner aux tuyaux de conduite ». If changes were made he asked to be kept informed so that he could adjust his theory accordingly. Furthermore, he recommended to undertake experiments on the thickness of the wall of the tubes and not to rely on extrapolations from other works on hydraulics, such as Mariotte's classic trea-

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[6] Euler to Frederick II, 20 April 1745. Berlin, Geheimes Staatsarchiv, I. HA Rep IX F2b fasz 15, 1730–1751, Acta betr. Gedruckte und dedicirte Bücher – Privilegia, Bl. 1–116, Bl. 61.

[7] C. A. Truesdell, "Rational Fluid Mechanics, 1687–1765"; Editor's introduction to *Euleri Opera Omnia*, II, 12, p. XXXVIII. (Euler's Collected Works, *Euleri Opera Omnia*, Series XX, Volume xx, are abbreviated in the following as *EOO*, XX, xx).

[8] Eduard Winter, ed., *Die Registres der Berliner Akademie der Wissenschaften 1746–1766. Dokumente für das Wirken Leonhard Eulers in Berlin* (Berlin: Akademie-Verlag, 1957), pp. 44, 62–63.

[9] Euler to Maupertuis, 21 September 1749. Reprinted in *EOO*, IV A, 6 (Basel: Birkhäuser, 1986), Doc. 54, pp. 136–137: "I take the liberty to send you my investigations on the Hydraulic Machine of Sans Soucy, which will be, I believe, sufficient to direct such a work, so that one may be assured that it will succeed. Would you please, Monsieur, have the goodness to present this package to the King, if you do not find anything in it which might prevent you from doing so..." Unless indicated otherwise, the translations are mine, M.E.

[10] Frederick II to Euler, 27 September 1749. *EOO*, IV A, 6, Doc. 18, p. 320.

tise on hydraulics that was based on his experiences in the Park of Versailles.[11] Euler recommended:

> Mais en cas que l'experience de Mariotte ne fut pas juste, ou gatée par une faute d'impression, je ne saurois rien determiner sur l'epaisseur des tuyaux dans le cas dont il s'agit, à moins qu'on ne fit de nouveau des experiences sur la force que des tuyaux de plomb sont capables de soutenir. Car on risqueroit trop si l'on vouloit confier au seul hasard la determination de l'epaisseur des tuyaux.[12]

It is remarkable that Euler assumed that metal tubes (« tuyaux de plomb ») would be used while Baumann and Heintze constructed the pipeline again with wooden tubes – despite the first failure in 1748 with barrel-like wooden tubes and against Euler's explicit advice to perform experiments on the strength of the pipes.

Euler continued to contribute his advice. On 23 October 1749 he presented a first treatise, *Sur le mouvement de l'eau par des tuyaux de conduite*,[13] to the Academy (see below), in which he presented the theory of non-stationary pipe-flow and determined the amount of water that could be raised using pumps of given powers and pipes of given dimensions. As he explained to Maupertuis two days before he presented his treatise at the Academy:

> La veritable cause de ce facheux accident consistoit uniquement en ce que la capacité des pompes étoit trop grande, et à moins qu'on ne la diminue très considerablement, ou en diminuant leur diametre ou leur hauteur, ou le nombre des jeux qui repond à un tour de moulin, la machine ne sera pas en etat de fournir une seule goute d'eau dans le reservoir.[14]

The day after the Academy meeting Euler wrote to Maupertuis, saying that he intended to present further treatises on this matter at later sessions.[15] He thereby alluded to two sequels, *Discussion plus particulière des diverses manières d'élever l'eau* and *Maximes pour arrenger le plus avantageusement les machines à élever l'eau par le moyen des pompes* (see below). Euler also presented to the King a summary of his researches on hydraulics together with related problems concerning windmills (which were presented

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[11] Edme Mariotte, *Traité du Mouvement des Eaux, Ve partie, IIe Discours, Oeuvres de Mariotte*, (La Haye 1740), t. II, p. 472.

[12] Euler to Maupertuis, 30 September 1749. *EOO*, IV A, 6, Doc. 55, pp. 137–138: "But in case Mariotte's experiment was not precise, or marred by a printing error, I would not know how to determine anything on the thickness of the tubes in the situation at hand, unless one would repeat the experiments on the force which tubes made of lead can sustain. For wanting to trust mere chance in determining the thickness of the tubes would be taking too much risk."

[13] Reprinted (and refered to as E 206) in *EOO*, II, 15, pp. 219–250.

[14] Euler to Maupertuis, 21 October 1749. *EOO*, IV A, 6, Doc. 56, pp. 139–140: "The true cause of this irksome accident consisted solely in that the capacity of the pumps was too large, and unless one reduces it considerably, by diminishing either their diameter, or their height, or the number of cycles corresponding to one turn of the mill, the machine will be unable to deliver one single drop of water into the reservoir."

[15] Euler to Maupertuis, 24 October 1749. *EOO*, IV A, 6, Doc. 57, pp. 140–141.

to the Academy in separate treatises),[16] warning once again that the scheme being used at Sanssouci for the fountain project was doomed to failure:

> Car si l'on ne vouloit rien changer de ce coté, il seroit presque impossible d'élever plus de 160 pied cubes par heure; pour cet effet il faudroit meme faire des changemens considerables dans les dimensions des pompes. Car sur le pied qu'elles se trouvent actuellement, il est bien certain, qu'on n'éleveroit jamais une goutte d'eau jusqu' au reservoir, et toute la force ne seroit employée qu'à la destruction de la machine et des tuyaux.[17]

Euler's theory of pipe-flow

There is no documentary evidence that Euler was involved with the further progress of the fountain project at Sanssouci beyond this exchange of letters in the autumn of 1749 and his presentation of the related treatises to the Berlin Academy. But a closer look at Euler's memoir on pipe-flow[18] provides clear evidence that the failure of the fountain project was not Euler's fault. Referring to Fig. 1, Euler outlined the problem as follows:

> Soit (Fig. l) ABCD le corps de pompe, dans lequel le piston joue, dont je suppose la cavité cylindrique, et on sait que le piston doit etre un cylindre de meme diametre pour remplir exactement la cavité de la pompe. Avec la pompe soit uni en bas DE le tuyau montant DEYZGH, par lequel l'eau est refoulee de la pompe, pour se degorger en haut à GH à gueule bée dans le reservoir J. Je suppose ce tuyau d'une figure quelconque, mais en sorte que ses sections faites perpendiculairement sur sa longueur soient partout circulaires, dont les diametres varient selon une raison quelconque par rapport aux divers endroits de ce tuyau.

The problem, then, is stated in a general manner:

> Le piston étant poussé en bas par une force donnée, trouver à chaque instant le mouvement de l'eau et la pression qu'elle exerce sur tous les points du tuyau.[19]

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[16] See E. 229 and E. 233 in *EOO*, II, 16.

[17] Euler to Frederick II, 17 October 1749, *EOO*, IV A, 6, Doc. 19, pp. 320–330, here p. 322: "For if one were unwilling to change anything in this regard, it would be almost impossible to raise more than 160 cubic feet per hour; even for this effect one would have to make considerable changes in the dimensions of the pumps. For in the situation in which they are at present, it is quite certain, that one never would raise one drop of water as far up as the reservoir, and the entire force would be employed only for the destruction of the machine and the tubes."

[18] In what follows I refer to E 206 in *EOO*, II, 15, pp. 219–250. This volume contains Euler's work on hydraulics. It was edited by Jakob Ackeret, a pioneer in modern fluid dynamics with considerable engineering experience in hydraulics; among Ackeret's papers, preserved in the *Archive of the Eidgenössische Technische Hochschule Zürich*, there are extensive notes concerning his editorial work on Euler's hydraulic theory (Hs 552).

[19] The figure is reproduced from E 206, p. 223: "Let (Fig. 1) ABCD be the body of the pump in which the piston is moving, which I assume to have a cylindrical cavity; and, as is well known, the piston must be cylindrical with the same diameter in order to fill out exactly the cavity of the pump. Connected with the pump at the lower end DE is the tube DEYZGH leading upward,

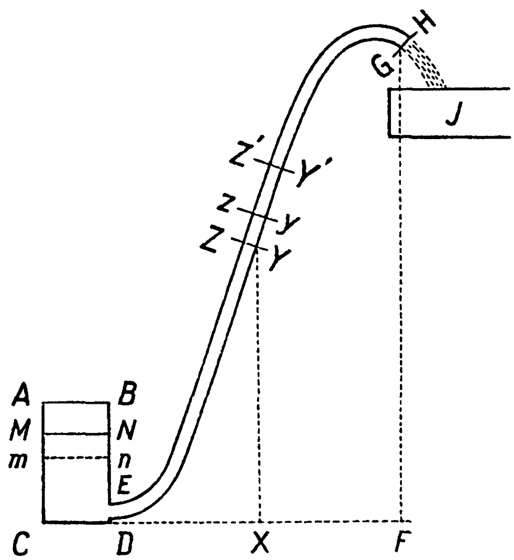

Figure 1

Euler assumes that the pump's piston is pushed down with a constant force K; he accounts for this force in terms of an equivalent water column of height k, and he derives the following formula for the pressure at y:[20]

$$p=(k-y)g\rho+(b-r)g\rho\left(1-\frac{w_p}{g}\frac{dw_p}{dr}\right)-a^2\rho w_p\frac{dw_p}{dr}\int\frac{ds}{z^2}+\frac{w_p^2}{2}\rho\left(1-\frac{a^4}{z^4}\right). \quad (1)$$

---

through which the water from the pump is pressed so that it flows out high up at GH through the open mouth into the reservoir J. I assume for this tube an arbitrary shape, but such that the perpendicular cross sections are everywhere circular along its length, with diameters varying in an arbitrary manner depending on the different positions along the tube"... "Assume the piston is pushed down by a given force: find for every instant the motion of the water and the pressure which it exerts on all points of the tube".

The following notation is used in the subsequent formulae: pump diameter AB = a; pump handle BE = b; pump piston distance at time t, AM = $r(t)$; increment distance in dt, Mm = dr; tube diameter at ZY = z; tube diameter at GH = h; height at ZY = y; distance DX = x; distance from ZY to zy = ds; height at GH = H.

[20] In Euler's original notation, the velocity of the piston (for which I use the symbol w_p in contrast to w for the velocity of the water in the pipeline) is expressed as $\sqrt{v}$, where v is an equivalent height of free fall. For an easier understanding I present Euler's original formulae with as little change as possible in a manner adjusted to our modern notation. Euler's notation is transformed into our notation by explicitely introducing the gravitational acceleration g (which is only implicit in Euler's notation, because Euler's units are given in equivalent free-fall heights). For more details on Euler's notation see Truesdell's editorial remarks in *EOO*, II, 12, p. XLIV, and Ackeret's introduction in *EOO*, II, 15, p. XX–XXI.

Here the integral is taken along the length of the pipeline from DE to YZ; we may write this as:

$$p + yg\rho + \frac{\rho}{2}w^2 + \rho\frac{d}{dt}\int wds = (k + b - r)g\rho + \frac{\rho}{2}w_p^2 - \rho\frac{dw_p}{dt}(b - r)$$

where we have substituted

$$\frac{dw_p}{dr} = \frac{dw_p}{dt}\frac{1}{dr\big/dt} = \frac{1}{w_p}\frac{dw_p}{dt} \quad \text{and} \quad \frac{a^2}{z^2}w_p = w$$

for the velocity in the tube. Reformulated in this way, it is evident that the right-hand side accounts for the pressure at the pump and the left-hand side for the pressure at an arbitrary position y along the tube; in differential form, this expression corresponds to the equation of motion for a fluid element along a streamline:

$$\frac{\partial}{\partial s}\left(p + yg\rho + \frac{\rho w^2}{2}\right) = -\rho\frac{\partial w}{\partial t}.$$

Trivial as these reformulations of Euler's result might appear from a modern perspective, where the equation of motion for a fluid element is an obvious starting point for hydrodynamical calculations, it is remarkable that Euler used this approach well before his famous general formulation of the equations of motion for fluids ("Euler's equations"), which he published in the proceedings of the Berlin Academy in 1755 in a memoir entitled *Principes généraux du mouvement des fluides.*[21] The argument by which Euler arrived at his formula above is the same as the one he used in his later work and the one that is still presented today,[22] that is, by balancing the forces on a fluid element, here in a cylindrical volume that is bounded by infinitely close cross sections separated by an infinitesimal pressure difference dp. It is this dynamical conception together with the notion of internal pressure that makes Euler's pipe-flow memoir so remarkable. Here "for the first time in the history of fluid mechanics, the *pressure p* in its modern sense has made its appearance" – so Truesdell emphasized the particular theoretical merit of Euler's procedure.[23] Also noteworthy is that Euler presented this procedure not in an abstract manner but with reference to a special case of practical importance.

Euler's practical work for the Sanssouci fountain thus was a prelude to his pioneering hydrodynamical contributions. If Euler had been interested in theoretical principles only, he could have ended his pipe-flow study at this point and proceeded to the more general case with which he dealt in his 1755 memoir. Instead, he started to explore the practical consequences of equation (1). Setting the pressure at the upper end of the pipeline equal to zero, Eq. (1) is transformed into a differential equation for the motion of the pump's piston, $w_p(r)$, or more conveniently for Euler's original quantity $v = v(r) = \frac{w_p^2(r)}{2g}$:

$$A\frac{dv}{dr} + Bv + C = 0, \tag{2}$$

[21] Reprinted as E 226 in *EOO*, II, 12, 54–91.

[22] See, for example, Hunter Rouse, *Elementary Mechanics of Fluids* (New York: Dover, 1978), pp. 45–47.

[23] Truesdell, *EOO*, II, 12, p. XLV.

with

$$A = b - r + a^2 \int_{DE}^{GH} \frac{ds}{z^2},$$

$$B = \frac{a^4}{h^4} - 1,$$

$$C = k + b - H.$$

(The integral in A may be calculated for any given tube diameter z along the length of the pipeline from DE to GH. Henceforth, we substitute $L = a^2 \int_{DE}^{GH} \frac{ds}{z^2}$. If the pipeline has a constant diameter h and length l, we have $L = \frac{a^2}{h^2} l$). Although this differential equation "soit absolument intégrable," as Euler observed, its solution would not be of great help "puisque l'expression devient si compliquée qu'on n'en sauroit tirer beaucoup de fruit".[24] Instead, Euler investigated the case when the elevation to which the water had to be raised was large compared to the dimensions of the pump, so that b and r could be neglected against H. In this approximation equation (2) could be written as[25]

$$dr = \frac{L dv}{k - H - v\left(\frac{a^4}{h^4} - 1\right)},$$

which yields

$$r = \frac{L}{\frac{a^4}{h^4} - 1} \ln \frac{k - H}{k - H - v\left(\frac{a^4}{h^4} - 1\right)} \tag{3}$$

Because for the Sanssouci problem the order of magnitude of the quantities was such that $k - H \gg v\left(\frac{a^4}{h^4} - 1\right)$, the logarithm can be approximated as $\frac{v}{k-H}\left(\frac{a^4}{h^4} - 1\right)$; we obtain $r = \frac{L}{k-H} v$, which after substituting $v = v(r) = \frac{w_p^2(r)}{2g}$ yields:

$$w_p^2 = \frac{2g(k - H)r}{L}$$

and

$$w_p \frac{dw_p}{dr} = \frac{g(k - H)}{L} \tag{4}$$

With these expressions the pressure at any point of the pipeline is obtained from Eq. (1):

---

[24] *EOO*, II, 15, p. 231: "may be integrated completely"... "because the expression becomes so complicated that one would hardly know how to draw much result from it."

[25] *EOO*, II, 15, p. 232.

$$
\begin{aligned}
p &= (k-y)g\rho + (b-r)g\rho\left(1-\frac{k-H}{L}\right) \\
&\quad -\rho\frac{g(k-H)}{L}a^2\int\frac{ds}{z^2} + \frac{g(k-H)}{L}\rho\left(1-\frac{a^4}{z^4}\right) \\
&= g\rho\left\{k-y+b-r+\frac{k-H}{L}\left(1-\frac{a^4}{z^4}-b+r\right)-\frac{(k-H)}{L}a^2\int\frac{ds}{z^2}\right\} \\
&\approx g\rho\left\{k-y-\frac{(k-H)}{L}a^2\int\frac{ds}{z^2}\right\}, \qquad (1^*)
\end{aligned}
$$

where in the last step we have neglected terms of the order of the piston dimensions against those of the order of the length and elevation of the pipeline.[26]
By integrating equation (4), the time ϑ required for one downward motion of the piston is obtained, which allows one to determine the amount of water raised per second:

$$
w_p = \frac{dr}{dt} = \sqrt{\frac{2g(k-H)r}{L}},
$$

or

$$
\int_0^{\vartheta} dt = \int_0^{b} \frac{dr}{\sqrt{\frac{2g(k-H)r}{L}}} = \sqrt{\frac{2bL}{g(k-H)}}.
$$

Within this time the piston sweeps over a volume of $\frac{1}{4}a^2b\pi$, so that the rate of flow of the water raised by the pump is

$$
Q = \frac{1}{4}a^2b\pi\frac{1}{\vartheta} = \frac{1}{4}a^2\pi\sqrt{\frac{gb(k-H)}{2L}} = \frac{a^2\pi}{8}\sqrt{\frac{2gbH}{L}}\sqrt{\frac{k}{H}-1}. \qquad (5)
$$

The effort exerted by the pump's piston per second (= work done per second during the downward motion of the pump's piston or the supplied power) is

$$
P_{input} = \frac{1}{4}a^2b\pi\rho gk\frac{1}{\vartheta} = \frac{1}{4}a^2\pi\rho gk\sqrt{\frac{gb(k-H)}{2L}} = \frac{a^2\pi\rho}{8}\sqrt{\frac{2gbH}{L}}gH\frac{k}{H}\sqrt{\frac{k}{H}-1}.
$$

With $\lambda = \frac{k}{H}$ as a dimensionless measure of the force exerted on the pump's piston, the most important consequences for the practical application of Euler's theory are:

1) The pressure at the lower end of the pipeline is $p \approx \rho gk = \rho gH\lambda = p_0\lambda$, where p_0 is the hydrostatic pressure due to the height H of the water column in the pipeline between the exit of the pump and the reservoir;
2) The output (work done by the raised water per second) is

[26] *EOO*, II, 15, p. 233.

$$P_{output} = \rho g H Q = gH\frac{a^2\pi\rho}{8}\sqrt{\frac{2gbH}{L}}\sqrt{\lambda - 1};$$

3) The input is $P_{input} = \frac{a^2\pi\rho}{8}\sqrt{\frac{2gbH}{L}}gH\lambda\sqrt{\lambda - 1}$, so the efficiency[27] of the water-raising machine is $P_{out}/P_{in} = 1/\lambda$.

In other words, the efficiency of the pump *decreases* with $1/\lambda$, while the amount of water raised per second is proportional to $\sqrt{\lambda - 1}$ and the pressure exerted against the tube at the lower end of the pipeline is proportional to λ.

The above reconstruction deviates from Euler's memoir for the sake of making his argument lucid in terms of *our* understanding. But Euler arrived at equivalent formulae and outlined their consequences for *his* contemporaries in the form of numerical examples, rules, and a table, so there is no doubt about his attempt to make his theory accessible for practical application.

For example, if the diameter of the pipeline is assumed to be constant, $z = c$, we have $L = \frac{a^2}{c^2}l$; by substituting in equation (5) we find $Q = \frac{ac\pi}{8}\sqrt{\frac{2gbH}{l}}\sqrt{\lambda - 1}$. Given the elevation of the reservoir (H), the force acting on the pump (k or λ) and the parameters of the pump (a and b), the only variables left for the practitioner to achieve the best performance are the diameter of the pipeline and its length: because $Q \propto c\sqrt{1/l}$, the pipeline should be as wide and as short as possible; because of $l \geq H$, "as short as possible" means that it should rise vertically to the reservoir – a conclusion that is counter-intuitive at first sight. Euler presented these results not merely by presenting the formulae but also verbally in the form of two rules, which he highlighted as "Regle 1" and "Regle 2":

> Pour que la meme force qui agit sur les pistons des pompes soit en état de fournir dans le réservoir la plus grande quantité d'eau, il faut avoir soin de faire le tuyau montant aussi large qu'il sera possible.
> Pour fournir une plus grande quantité d'eau dans le réservoir par la meme force qui agit sur les pistons, il faut rendre le tuyau montant aussi court qu'il sera possible. Par conséquent, comme la hauteur du réservoir est donnée, if faut faire le tuyau montant non seulement droit, mais aussi sa direction perpendiculaire, autant que les circonstances le permettent.[28]

In case more than these quantities were subject to modification, Euler presented his results in tabular form, depending on a range of λ's: From this table a practitioner would be able to determine, for example, the diameter of the cylinder of the pump appropriate

[27] Euler did not introduce this quantity, but he was aware how the performance of the pump changed with the increase of the force exerted on the piston. See *EOO*, II, 15, p. 243–245, and Ackeret's introductory remarks in LIII-LV.

[28] *EOO*, II, 15, pp. 240–242: Rule 1: "In order to enable the same force, acting on the pistons of the pumps, to deliver the largest quantity of water to the reservoir, one must take care to make the rising tube as wide as possible." Rule 2: "In order to deliver the largest amount of water to the reservoir by the same force acting on the pistons, one has to make the rising tube as short as possible. Consequently, because the height of the reservoir is given, one has to make the rising tube not only straight, but also its direction perpendicular, as far as the circumstances permit."

for a range of forces and a given upper limit of the pressure that the pipeline tubes could withstand: « un cas qui a fort souvent lieu, puisqu'il n'est pas toujours possible de rendre les tuyaux montans aussi forts qu'on veut; et partant il faut regler dans ces cas les autres parties de la machine, en sorte qu'on n'ait rien a craindre du coté de la force des tuyaux. »[29]

In particular, Euler was very outspoken with regard to the high pressure at the lower end of the pipeline, which could be much higher than what one would expect from the hydrostatic pressure due to the height of the water column in the pipeline. At the end of his memoir he presented a numerical example to illustrate this: for a pump with two pistons with a "jeu" (ϑ) of 6 seconds, $a = \frac{4}{3}$ feet, $b = 4$ feet, connected to a pipeline of $c = \frac{3}{4}$ feet, $l = 3000$ feet, $H = 60$ feet, Euler found for the pressure at the lower end of the pipeline a value corresponding to a 330-foot high column of water:

> Donc, si le tuyau n'avoit pas été assés fort pour porter une colomne d'eau de 330 pieds de hauteur, il seroit crevé infalliblement; quoique la hauteur de l'élévation de l'eau ne fut que de 60 pieds, de sorte que le tuyau dut soutenir une force plus de 5 fois plus grande que le simple poids de la colomne d'eau.[30]

Theory and practice

Euler's pipe-flow treatise extends over 37 printed pages, most of it aiming at practical applications. In two sequels, his 35-page memoir, *Discussion plus particulière des diverses manières d'élever l'eau*, and his 47-page memoir, *Maximes pour arranger le plus avantageusement les machines à élever l'eau par le moyen des pompes*, he concentrates in more detail on pump problems and elaborates on their practical consequences in a popular manner – to the extent that the editor of these treatises criticized their tabular presentations "even for very elementary formulae" and "ponderous repetitions."[31]

Despite its practical goals, however, Euler's theory hardly could have served as a blueprint for the design of the tubes, pumps, and other parts of the hydraulic installations at Sanssouci. Euler was realistic enough not to pretend that it could serve such a purpose. For example, in his letters to Maupertuis, quoted above, he made clear that it was necessary to perform "des experiences sur la force que des tuyaux de plomb sont capables de soutenir."[32] This suggestion must have been in the back of the King's mind when he reproached Baumann and Heintze for not having undertaken experiments before they built the entire pipeline with inappropriate materials. Even today a theorist would not be able

[29] *EOO*, II, 15, p. 245: "a case which happens very often, because it is not always possible to make the rising tubes as strong as one wishes; hence in such cases one has to modify the other parts of the machine, in order that nothing remains to be afraid of with respect to the force of the tubes."

[30] *EOO*, II, 15, pp. 249–250: "Therefore, if the tube had not been strong enough to carry a 330 feet high water column, it would have burst inevitably; even although the height for raising the water was only 60 feet, the tube would have had to withstand a resulting force more than 5 times larger than the simple weight of the water column."

[31] Ackeret, *EOO*, II, 15, p. LVI.

[32] Euler to Maupertuis, 30 September 1749. *EOO*, IV A, 6, Doc. 55, pp. 137–138.

to predict the strength of materials from first principles only. Euler had suggested to use lead – not for theoretical reasons but because of contemporary experience, such as described in Bernard Forest de Bélidor's *Architecture hydraulique*,[33] which was published ten years before the Sanssouci project was begun and which was explicitly mentioned in Euler's letter to Maupertuis. The practitioners, however, ignored Euler's suggestions and all other contemporary expert knowledge as published in Bélidor's classic work.

A similar remark should be made about Euler's neglect of friction in his pipe-flow memoir. No hydrodynamic theory was (and to some extent even today is) able to predict frictional effects in pipes from first principles. The formulation of the equations of motion that included viscosity, the Navier-Stokes equations, was almost a hundred years in the future. But even knowledge of these equations would not have enabled one to account for friction quantitatively, so that generations of 19th and 20th-century hydraulic engineers developed their own body of knowledge to cope in practice with their projects.[34]

With theoretical hydrodynamics so different from practical hydraulics, we can ask if Euler's theory was altogether futile? If so, we would be tempted to conclude that it is not surprising that it was ignored by the practitioners at Sanssouci. But in view of the bungling at Sanssouci we have to conclude that these practitioners also were not acting according to contemporary empirical knowledge in hydraulics. What seems futile for a 20th-century hydraulic engineer, because Euler's theory expounds only fundamental principles, could well have provided useful suggestions for an 18th-century "fontainier," because Euler's theory pointed to the crucial problems associated with the Sanssouci project. From a modern perspective, for example, it is rather trivial to understand why extremely high pressures in a pipeline may result from the acceleration of the fluid mass, but such dynamic thinking in fluid mechanics was unknown before Euler's theory, when it was believed that the height of the water column in the pipeline determined the force against its wall.

It is a myth that Newton's laws of motion were sufficient to understand any problem in mechanics, be it in statics or dynamics, as Ernst Mach asserted in his classic history of mechanics – and as was criticized for doing so by writers experienced in continuum mechanics.[35] The same applies to the principle of the conservation of energy in fluid mechanics: Just as Newton's second law does not by itself imply Euler's equation for the motion of a fluid (based on the understanding of internal pressure), so Leibniz's concept of the balance of mechanical energies does not by itself imply the Bernoulli equation in fluid mechanics. As we saw in the introduction, Armin Hermann argued that Euler should have consulted Leibniz's writings for the Sanssouci problem. But why should he have done so? Was it not Euler who provided us with our modern understanding

[33] Bernard Forest de Bélidor, *Architecture hydraulique ou l'art de conduire, d'elever et de ménager les eaux pour les differentes besoins de la vie*, 2 vols. (Paris: 1737–1739).

[34] Hunter Rouse, Simon Ince, *History of Hydraulics*. (Ames: State University of Iowa, Iowa Institute of Hydraulic Research, 1957).

[35] István Szabó, „Bemerkungen zur Literatur über die Geschichte der Mechanik," *Humanismus und Technik, 22:3* (1979), 121–154. For Truesdell, Mach served as an example for a „competent scientist but crudely biased and historically ignorant person." C. A. Truesdell, *An Idiot's Fugitive Essays on Science* (New York: Springer, 1984), p. 448.

of the Bernoulli equation? Was he not the first to make use of the non-stationary Bernoulli equation in his pipe-flow theory? What could he possibly have found in Leibniz's treatises that would have brought him more insight into the problem?

Even more astonishing is the statement that Euler was only a second-rate physicist. After their extensive study of Euler's work, it is only fair to leave it to the editors of Euler's *Opera Omnia* volumes on hydrodynamics (Truesdell) and hydraulics (Ackeret) to comment on this reproach: "While Euler is known today primarily as a mathematician, he was also the greatest physicist of his era," Truesdell wrote in a biographical essay.[36] Truesdell does not refer to hydrodynamics only: With regard to Euler's works on naval science and artillery, Condorcet asserted that these researches "have been of almost no use except to the science of analysis"[37] – to which "still current slander" Truesdell responded: "If indeed Euler's work on ships and projectiles was of no practical use, it was only because the 'practical' men chose not to notice it."[38] Ackeret, too, found much evidence in Euler's memoirs to praise his sense of practical physics. As an academic fluid dynamicist and engineer who worked for a few years in the turbine industry, Ackeret was particularly interested in Euler's theory of reaction water wheels. He designed a small turbine according to Euler's specifications that came close to the efficiency of modern turbines.[39] He found that Euler was the first to notice the problem of cavitation – 150 years before this phenomenon was recognized as a major problem in hydraulic engineering.[40] With regard to Euler's pipe-flow theory, Ackeret concluded that "there are important practical remarks, which demonstrate what good feel Euler had for practical problems."[41]

Who then is to blame for the failure of the fountain project at Sanssouci? We should compare the Sanssouci effort with other hydraulic projects of the 17th and 18th centuries before answering this question. The most famous hydraulic project was the "machine de Marly," a monstrous installation on the Seine river, constructed between 1681 and 1688, by which water for the fountains in the Park of Versailles was raised over three stages to a level of about 150 meters above the Seine, from where it was conducted to a reservoir 37 meters above the level of the fountains at the castle. To raise the water not in one step but to intermediate levels was a means of avoiding high pressure in the lower part of the pipeline.[42] The force to operate the pumps at the intermediate levels was transmitted by

[36] Truesdell (n. 35), p. 341.

[37] Quoted from Condorcet's „Éloge de M. Euler" in Truesdell, *EOO*, II, 12, p. XLVI.

[38] *Ibid.* See also: C. Truesdell, "Euler's Contribution to the Theory of Ships and Mechanics. An Essay Review," *Centaurus, 26*, (1983), 323–335.

[39] Jakob Ackeret, „Untersuchung einer nach den Eulerschen Vorschlägen (1754) gebauten Wasserturbine," *Schweizerische Bauzeitung, 123* (1944) 9–15.

[40] Ackeret, *EOO*, II, 15, p. VII. See also Ackeret's unpublished lecture on „Leonhard Euler's Arbeiten zur Maschinentechnik," *Archive of the Eidgenössische Technische Hochschule Zürich*, HS 552:144.

[41] Ackeret, *EOO*, II, 15, 15, p. LVI.

[42] Wolfhard Weber, „Marly – ein Schnittpunkt für wen?" in Günther Bayerl and Wolfhard Weber (eds.), *Sozialgeschichte der Technik* (Münster: Waxmann, 1998), 111–120. Weber argues on p. 114 that the „water pressure of 15 bar" would have been too high for the pipeline. This demonstrates that even a modern historian of technology has the same 18th-century misconception,

long rods (Stangenkunst) from the water wheels at the lowest level. The pipelines were made of tubes of cast iron. The entire installation was the result of practical experience, mainly that of a talented mechanic (Rennequin Sualem), who had been entrusted with the task after his employer had won a public contest to carry out the project.

A second example: More than a hundred years before the Sanssouci project there were successful water-raising hydraulic installations in operation, such as a brine pipeline from Reichenhall to Traunstein, built between 1617 and 1619 across hilly country with height differences of hundreds of meters. Already then it was common practice to use cast iron and lead whenever the tubes had to withstand high pressures.[43] A third installation that could have served as an example was built near Munich and described in great detail in Bélidor's *Architecture hydraulique*: « une fort belle machine, exécutée à Nynphenbourg par M. le Comte de Wahl, Directeur des Batiments de l'Electeur de Baviere; son objet est d'élever l'eau à 60 pieds dans un réservoir, pour la faire jaillir dans le jardin électoral. »[44]

As is evident from these examples, the construction of water-raising machinery in the 18th century was a matter of practical expertise. The men entrusted with this task in the Park of Sanssouci did not possess such expertise; otherwise they would not have begun with wooden tubes for the pipeline ascending to the reservoir. Such tubes were in use for the conduction of water in pipes where no considerable pressure was to be expected, but in all cases of water-raising machines where the pipelines were subjected to high pressure, an experienced "fontainier" would have chosen metal pipes at the outset – as Euler had assumed they would do.

Origins of the Euler legend

Such installations were expensive. Rather than blaming Euler, a much more plausible cause of the bungling in the Park of Sanssouci was the King's stinginess, which was well known to his architects, who had to cope with his extravagant wishes on the one hand (such as a fountain jet to a height of 100 feet) and his stinginess with funds to realize them on the other. Manger cites many examples where the King's stinginess prevented efficient results, and he concluded his history of the constructions at Potsdam in despair: "Economizing is a virtue for everyone; but if it is exaggerated, it loses its meaning; and nowhere is exaggerated economizing so damaging as with constructions."[45]

Furthermore, the King had no understanding of technology, science, and mathematics. Although he wished to be surrounded by Europe's scientific celebrities, his personal

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that it is only height difference with its hydrostatic pressure (15 bar corresponds to 150 meters) that matters for the pressure in the pipeline at the lower end.

[43] Heinrich Kurtz, *Die Soleleitung von Reichenhall nach Traunstein 1617–1619*. (München: Deutsches Museum, 1978), published in *Abhandlungen und Berichte*, *46:1/2*, (1978).

[44] Bélidor, *Architecture hydraulique*, II, part 1, no. 986: "a very beautiful machine, constructed at Nymphenburg by M. le Comte de Wahl, director of buildings of the Elector of Bavaria; its objective is to raise water 60 feet high in a reservoir, so that it forms a fountain jet in the electoral garden." The original has the spelling "Nynphenbourg."

[45] Manger, *Baugeschichte*, Vol. 3, p. 547. With regard to the fountain project, see also Manger's comments on the King's stinginess in Vol. 2, pp. 270–272.

preference was for art and philosophy rather than mathematics.[46] His ideal were men who radiated French *ésprit*, such as d'Alembert, to whom he offered the presidency of the Berlin Academy after Maupertuis's death, not to Euler, who was not eloquent enough but who was intellectually superior to both Maupertuis and d'Alembert. Since his arrival in Berlin, Euler had hoped for the presidency. But even when d'Alembert declined and recommended Euler in his place, the King did not trust Euler with the presidency but instead assumed this office himself. After these depressing experiences Euler left Berlin and returned to St. Petersburg in 1766.[47]

Against this background we must evaluate the historical source upon which the reproach of Euler's failure at Sanssouci is based, namely Frederick the Great's letter to Voltaire. The passage reads in detail:

> Je voulus faire un jet d'eau en mon Jardin; le Ciclope Euler calcula l'éffort des roues, pour faire monter l'eau dans un bassin, d'ou elle devoit retomber par des canaux, afin de jaillir à Sans-Souci. Mon Moulin a été éxécuté géométriquement, et il n'a pu élever une goutte d'eau à Cinquante pas du Bassin. Vanité des Vanités ! Vanité de la géométrie.[48]

Here, in a nutshell, we have all of the ingredients of the legend: First, Frederick derides "le Ciclope Euler" (Euler had lost an eye in 1735) and has no understanding of the nature of the problem. Second, he ironically and almost literally repeats Euler's prophesy that one would not be able to raise "une goutte d'eau" to the reservoir,[49] but what Euler meant as a warning against the bungling at Sanssouci is formulated by the King as a reproach against mathematics. Finally, when Klemm (see the introduction) only quoted the last sentence in this passage as "Il n'a pu... "[50] instead of "et il n'a pu...," he added insult to insult, because now one was led to believe that "Il" is a reference to Euler instead of the "Moulin." But these subtleties only add another facet to the thoroughly distorted image that Euler, the mathematician, was a second-rate physicist and was remote from practical applications.

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[46] On the King's contempt for mathematics in general and Euler in particular, see Otto Spiess, *Leonhard Euler. Ein Beitrag zur Geistesgeschichte des XVIII. Jahrhunderts* (Leipzig: Huber und Co., 1929), p. 175.

[47] Eduard Winter, ed., *Die Registres der Berliner Akademie der Wissenschaften 1746–1766. Dokumente für das Wirken Eulers in Berlin* (Berlin: Akademie-Verlag, 1957), p. 83.

[48] Theodore Besterman, ed., *The Complete Works of Voltaire, Vol. 129: Correspondence and related documents, XLV September 1777-May 1778, letters D20780-D21221.* (Banbury: The Voltaire Foundation, 1976). D21010, Frederick II to Voltaire, 25 January 1778. pp. 184–186, here p. 185: "I wanted to make a jet of water in my Garden; the Cyclop Euler calculated the effort of the wheels for raising the water to a basin, from where it should fall down through canals, in order to form a fountain jet at Sans-Souci. My mill was constructed mathematically, and it could not raise one drop of water to a distance of fifty feet from the basin. Vanity of Vanities! Vanity of mathematics."

[49] Euler to Frederick II, 17 October 1749, *EOO*, IV A, 6, Doc. 19, pp. 320–330, here p. 322.

[50] Klemm (ref.3), p. 201.

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